UNIT – I: Transmission Lines - I

Types,Transmission Lineparameters,Transmission Line Equations,Primary and Secondary Constants, infinite line, Characteristic Impedance, Attenuation constant ,Phase shift constant, Propagation Constant, Phase and Group Velocities, Wave length.

LEARNING OBJECTIVES

Upon completion of this UNIT, you will be able to:

1. State what a transmission line is and how transmission lines are used.

2. Explain the operating principles of transmission lines.

3. Describe the types of transmission lines.

4. . Explain the theory of the transmission line.

6. Define the term LUMPED CONSTANTS in relation to a transmission line.

7. Define the term DISTRIBUTED CONSTANTS in relation to a transmission line.

8. Define the term CHARACTERISTIC IMPEDANCE

Measurable Student Learning Outcomes:

At the completion of the course(UNIT-I), students will be able to…

1. Identify the characteristics of transmission lines and transmission line circuits.

2. Analyze transmission line circuits.

INTRODUCTION TO TRANSMISSION LINES

A TRANSMISSION LINE is a device designed to guide electrical energy from one point to another. It is used, for example, to transfer the output rf energy of a transmitter to an antenna. This energy will not travel through normal electrical wire without great losses. Although the antenna can be connected directly to the transmitter, the antenna is usually located some distance away from the transmitter. On board ship,the transmitter is located inside a radio room and its associated antenna is mounted on a mast. A transmission line is used to connect the transmitter and the antenna.

The transmission line has a single purpose for both the transmitter and the antenna. This purpose is to transfer the energy output of the transmitter to the antenna with the least possible power loss. How well this is done depends on the special physical and electrical characteristics (impedance and resistance) of the transmission line.

TERMINOLOGY

All transmission lines have two ends (see figure 1-1). The end of a two-wire transmission line connected to a source is ordinarily called the INPUT END or the GENERATOR END. Other names given to this end are TRANSMITTER END, SENDING END, and SOURCE. The other end of the line is called the OUTPUT END or RECEIVING END. Other names given to the output end are LOAD END and SINK

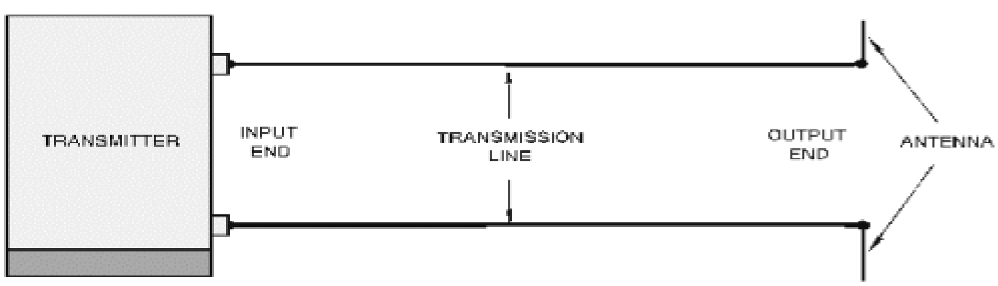


Figure 1-1 Basic transmission line

You can describe a transmission line in terms of its impedance. The ratio of voltage to current (E_{in}/I_{in}) at the input end is known as the INPUT IMPEDANCE (Z_{in}). This is the impedance presented to the transmitter by the transmission line and its load, the antenna. The ratio of voltage to current at the output ($E_{out}/I_{ou}t$) end is known as the OUTPUT IMPEDANCE (Z_{out}). This is the impedance presented to the load by the transmission line and its source. If an infinitely long transmission line could be used, the ratio of voltage to current at any point on that transmission line would be some particular value of impedance. This impedance is known as the CHARACTERISTIC IMPEDANCE.

TYPES OF TRANSMISSION MEDIUMS

The different types of TRANSMISSION MEDIUMS in electronic applications.

Each medium (line or wave guide) has a certain characteristic impedance value, current-carrying capacity,and physical shape and is designed to meet a particular requirement.

The five types of transmission mediums that we will discuss in this chapter include PARALLEL-LINE, TWISTED PAIR, SHIELDED PAIR, COAXIAL LINE, and WAVEGUIDES. The use of a particular line depends, among other things, on the applied frequency, the power-handling capabilities, and the type of installation.

Two-Wire Open Line

One type of parallel line is the TWO-WIRE OPEN LINE illustrated in figure 1-2. This line consists of two wires that are generally spaced from 2 to 6 inches apart by insulating spacers. This type of line is most often used for power lines, rural telephone lines, and telegraph lines. It is sometimes used as a transmission line between a transmitter and an antenna or between an antenna and a receiver. An advantage of this type of line is its simple construction. The principal advantages of this type of line are the high radiation losses and electrical noise pickup because of the lack of shielding. Radiation losses are produced by the changing fields created by the changing current in each conductor.

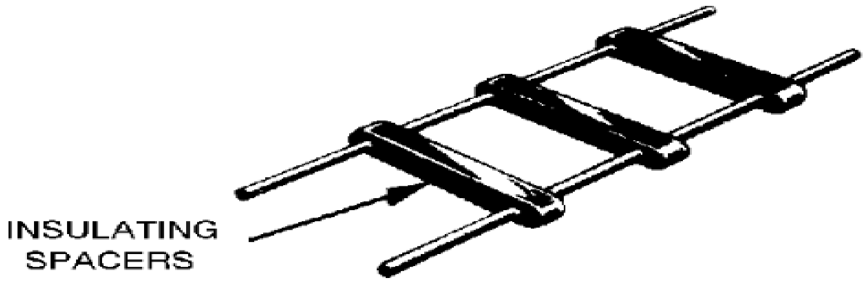


Figure 1-2.Parallel two-wire line.

Another type of parallel line is the TWO-WIRE RIBBON (TWIN LEAD) illustrated in figure1-3.This type of transmission line is commonly used to connect a television receiving antenna to a home television set. This line is essentially the same as the two-wire open line except that uniform spacing is assured by embedding the two wires in a low-loss dielectric, usually polyethylene. Since the wires are embedded in the thin ribbon of polyethylene, the dielectric space is partly air and partly polyethylene.

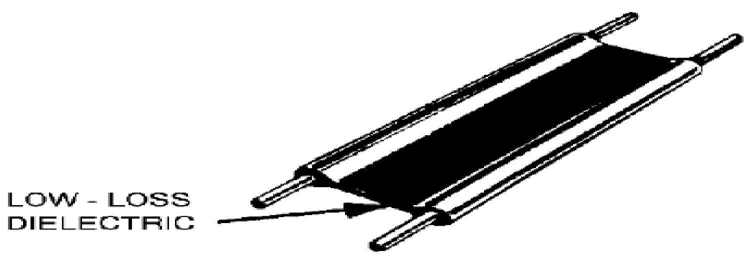


Figure 1-3.—Two-wire ribbon type line.

Twisted Pair

The TWISTED PAIR transmission line is illustrated in figure 1-4. As the name implies, the line consists of two insulated wires twisted together to form a flexible line without the use of spacers. It is not used for transmitting high frequency because of the high dielectric losses that occur in the rubber insulation. When the line is wet, the losses increase greatly.

Figure 1-4 Twisted pair.

Shielded Pair

The SHIELDED PAIR, shown in figure 1-5, consists of parallel conductors separated from each other and surrounded by a solid dielectric. The conductors are contained within a braided copper tubing that acts as an electrical shield. The assembly is covered with a rubber or flexible composition coating that protects the line from moisture and mechanical damage. Outwardly, it looks much like the power cord of a washing machine or refrigerator.

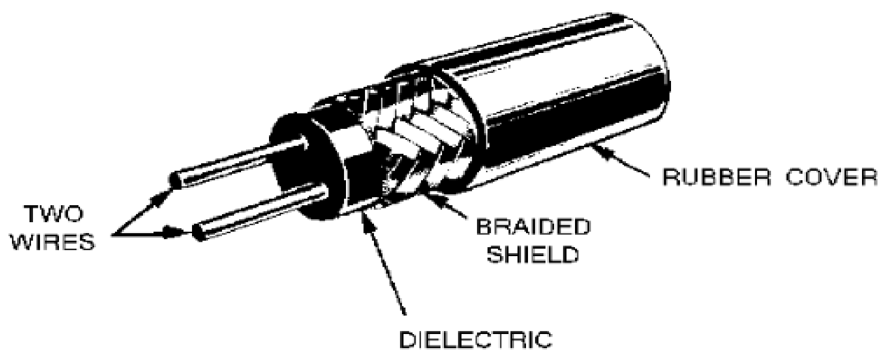


Figure 1-5.—Shielded pair.

The principal advantage of the shielded pair is that the conductors are balanced to ground; that is, the capacitance between the wires is uniform throughout the length of the line. This balance is due to the uniform spacing of the grounded shield that surrounds the wires along their entire length. The braided copper shield isolates the conductors from stray magnetic fields.

Coaxial Lines

There are two types of COAXIAL LINES, RIGID (AIR) COAXIAL LINE and FLEXIBLE (SOLID) COAXIAL LINE. The physical construction of both types is basically the same; that is, each contains two concentric conductors.

The rigid coaxial line consists of a central, insulated wire (inner conductor) mounted inside a tubular outer conductor. This line is shown in figure 3-6. In some applications, the inner conductor is also tubular.
The inner conductor is insulated from the outer conductor by insulating spacers or beads at regular intervals. The spacers are made of Pyrex, polystyrene, or some other material that has good insulating characteristics and low dielectric losses at high frequencies.

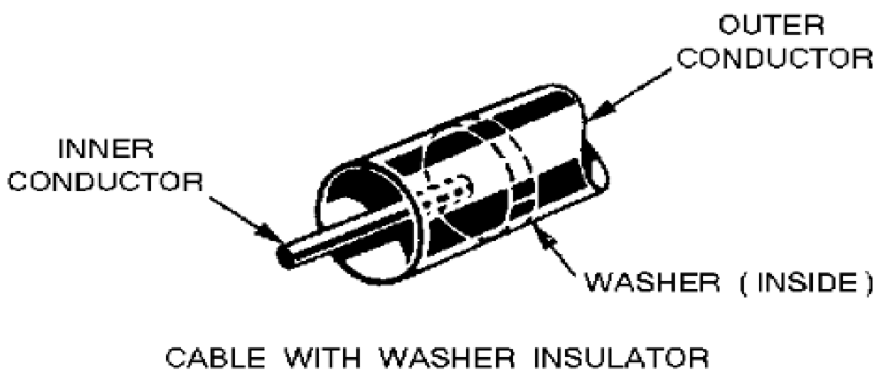


Figure 1-6 Air coaxial line.

The chief advantage of the rigid line is its ability to minimize radiation losses. The electric and magnetic fields in a two-wire parallel line extend into space for relatively great distances and radiation losses occur. However, in a coaxial line no electric or magnetic fields extend outside of the outer conductor. The fields are confined to the space between the two conductors, resulting in a perfectly shielded coaxial line. Another advantage is that interference from other lines is reduced.

The rigid line has the following disadvantages: (1) it is expensive to construct; (2) it must be kept dry to prevent excessive leakage between the two conductors; and (3) although high-frequency losses are somewhat less than in previously mentioned lines, they are still excessive enough to limit the practical length of the line.

Leakage caused by the condensation of moisture is prevented in some rigid line applications by the use of an inert gas, such as nitrogen, helium, or argon. It is pumped into the dielectric space of the line at a pressure that can vary from 3 to 35 pounds per square inch. The inert gas is used to dry the line when it is first installed and pressure is maintained to ensure that no moisture enters the line.

Flexible coaxial lines (figure 1-7) are made with an inner conductor that consists of flexible wire insulated from the outer conductor by a solid, continuous insulating material. The outer conductor is made of metal braid, which gives the line flexibility. Early attempts at gaining flexibility involved using rubber insulators between the two conductors. However, the rubber insulators caused excessive losses at high frequencies.

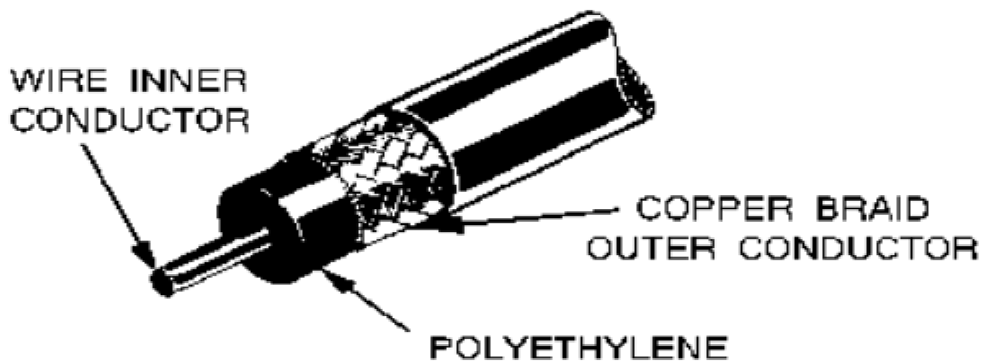


Figure 1-7 Flexible coaxial line

Because of the high-frequency losses associated with rubber insulators, polyethylene plastic was developed to replace rubber and eliminate these losses. Polyethylene plastic is a solid substance that remains flexible over a wide range of temperatures. It is unaffected by seawater, gasoline, oil, and most other liquids that may be found aboard ship. The use of polyethylene as an insulator results in greater high-frequency losses than the use of air as an insulator. However, these losses are still lower than the losses associated with most other solid dielectric materials.

Waveguides

The WAVEGUIDE is classified as a transmission line. However, the method by which it transmits energy down its length differs from the conventional methods. Waveguides are cylindrical, elliptical, or rectangular (cylindrical and rectangular shapes are shown in figure 1-8). The rectangular waveguide is used more requently than the cylindrical waveguide.

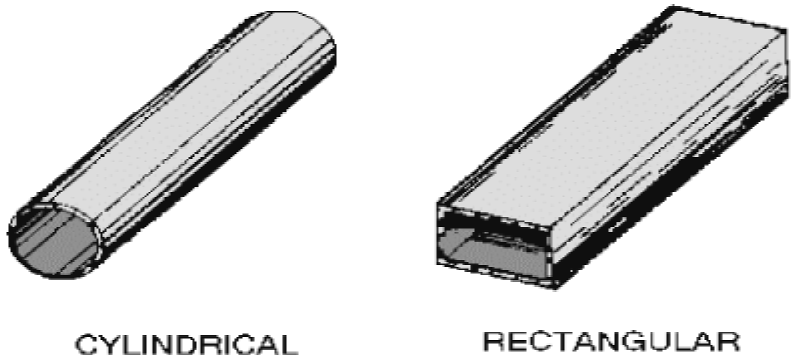


Figure 1-8.—Waveguides.

The term waveguide can be applied to all types of transmission lines in the sense that they are all used to guide energy from one point to another. However, usage has generally limited the term to mean a hollow metal tube or a dielectric transmission line. In this chapter, we use the term waveguide only to mean "hollow metal tube." It is interesting to note that the transmission of electromagnetic energy along a waveguide travels at a velocity somewhat slower than electromagnetic energy traveling through free space.

A waveguide may be classified according to its cross section (rectangular, elliptical, or circular), or according to the material used in its construction (metallic or dielectric). Dielectric waveguides are seldom used because the dielectric losses for all known dielectric materials are too great to transfer the electric and magnetic fields efficiently.

The installation of a complete waveguide transmission system is somewhat more difficult than the installation of other types of transmission lines. The radius of bends in the waveguide must measure greater than two wavelengths at the operating frequency of the equipment to avoid excessive attenuation.The cross section must remain uniform around the bend. These requirements hamper installation in confined spaces. If the waveguide is dented, or if solder is permitted to run inside the joints, the attenuation of the line is greatly increased. Dents and obstructions in the waveguide also reduce its breakdown voltage, thus limiting the waveguide’s power-handling capability because of possible arc over.Great care must be exercised during installation; one or two carelessly made joints can seriously inhibit the advantage of using the waveguide.

LENGTH OF A TRANSMISSION LINE

A transmission line is considered to be electrically short when its physical length is short compared to a quarter- wavelength of the energy it is to carry.

A transmission line is electrically long when its physical length is long compared to a quarter-wavelength of the energy it is to carry. You must understand that the terms "short" and "long" are relative ones. For example, a line that has a physical length of 3 meters (approximately 10 feet) is considered quite short electrically if it transmits a radio frequency of 30 kilohertz. On the other hand, the same transmission line is considered electrically long if it transmits a frequency of 30,000 megahertz.

When power is applied to a very short transmission line, practically all of it reaches the load at the output end of the line. This very short transmission line is usually considered to have practically no electrical properties of its own, except for a small amount of resistance.

However, the picture changes considerably when a long line is used. Since most transmission lines are electrically long (because of the distance from transmitter to antenna), the properties of such lines must be considered. Frequently, the voltage necessary to drive a current through a long line is considerably greater than the amount that can be accounted for by the impedance of the load in series with the resistance of the line.

TRANSMISSION LINE THEORY

The electrical characteristics of a two-wire transmission line depend primarily on the construction of the line. The two-wire line acts like a long capacitor. The change of its capacitive reactance is noticeable as the frequency applied to it is changed. Since the long conductors have a magnetic field about them when electrical energy is being passed through them, they also exhibit the properties of inductance. The values of inductance and capacitance presented depend on the various physical factors that we discussed earlier. For example, the type of line used, the dielectric in the line, and the length of the line must be considered. The effects of the inductive and capacitive reactances of the line depend on the frequency applied. Since no dielectric is perfect, electrons manage to move from one conductor to the other through the dielectric. Each type of two-wire transmission line also has a conductance value. This conductance value represents the value of the current flow that may be expected through the insulation. If the line is uniform (all values equal at each unit length), then one small section of the line may represent several feet. This illustration of a two-wire transmission line will be used throughout the discussion of transmission lines; but, keep in mind that the principles presented apply to all transmission lines. We will explain the theories using LUMPED CONSTANTS and DISTRIBUTED CONSTANTS to further simplify these principles.

LUMPED CONSTANTS

A transmission line has the properties of inductance, capacitance, and resistance just as the more conventional circuits have. Usually, however, the constants in conventional circuits are lumped into a single device or component. For example, a coil of wire has the property of inductance. When a certain amount of inductance is needed in a circuit, a coil of the proper dimensions is inserted. The inductance of the circuit is lumped into the one component. Two metal plates separated by a small space, can be used to supply the required capacitance for a circuit. In such a case, most of the capacitance of the circuit is lumped into this one component. Similarly, a fixed resistor can be used to supply a certain value of circuit resistance as a lumped sum. Ideally, a transmission line would also have its constants of inductance,capacitance, and resistance lumped together, as shown in figure 1-9. Unfortunately, this is not the case.Transmission line constants are distributed , as described below.

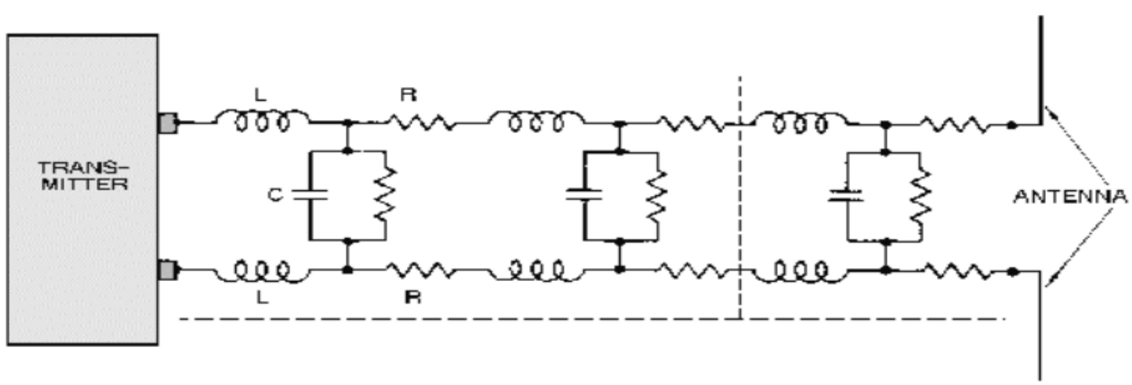


Figure 1-9 .—Equivalent circuit of a two-wire transmission line.

DISTRIBUTED CONSTANTS

Transmission line constants, called distributed constants, are spread along the entire length of the transmission line and cannot be distinguished separately. The amount of inductance, capacitance, and resistance depends on the length of the line, the size of the conducting wires, the spacing between the wires, and the dielectric (air or insulating medium) between the wires. The following paragraphs will be useful to you as you study distributed constants on a transmission line.

Inductance of a Transmission Line

When current flows through a wire, magnetic lines of force are set up around the wire. As the current increases and decreases in amplitude, the field around the wire expands and collapses accordingly. The energy produced by the magnetic lines of force collapsing back into the wire tends to keep the current flowing in the same direction. This represents a certain amount of inductance, which is expressed in microhenrys per unit length . Figure 1-10 illustrates the inductance and magnetic fields of a transmission line.

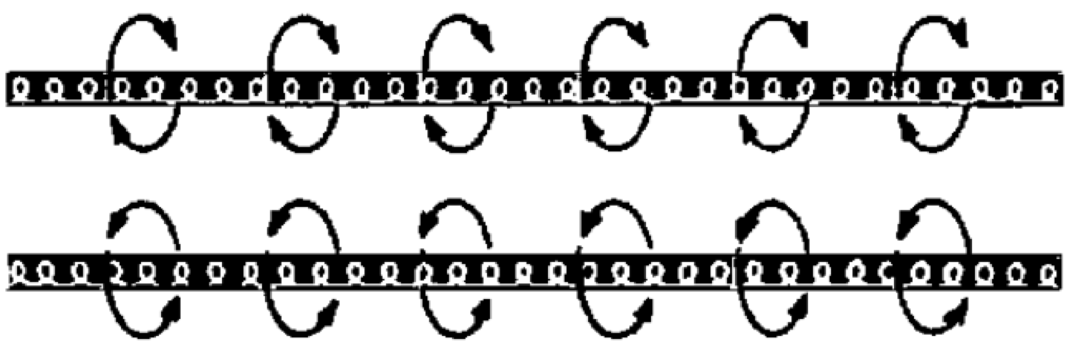


Figure 1-10.—Distributed inductance

Capacitance of a Transmission Line

Capacitance also exists between the transmission line wires, as illustrated in figure 1-11. Notice that the two parallel wires act as plates of a capacitor and that the air between them acts as a dielectric. The capacitance between the wires is usually expressed in picofarads per unit length . This electric field between the wires is similar to the field that exists between the two plates of a capacitor.

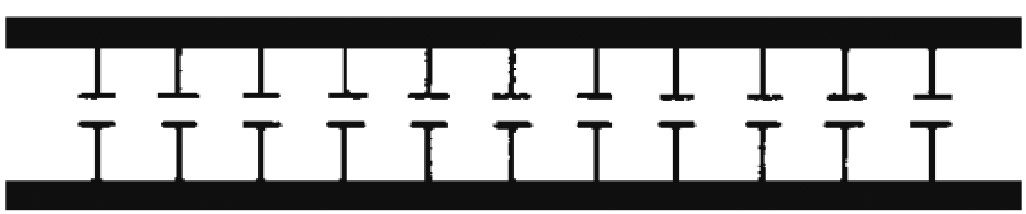


Figure 1-11.—Distributed capacitance.

Resistance of a Transmission Line

The transmission line shown in figure 1-12 has electrical resistance along its length. This resistance is usually expressed in ohms per unit length and is shown as existing continuously from one end of the line to the other.

Figure 1-12.—Distributed resistance.

Leakage Current

Since any dielectric, even air, is not a perfect insulator, a small current known as LEAKAGECURRENT flows between the two wires. In effect, the insulator acts as a resistor, permitting current to pass between the two wires. Figure 1-13 shows this leakage path as resistors in parallel connected between the two lines. This property is called CONDUCTANCE (G) and is the opposite of resistance.

Conductance in transmission lines is expressed as the reciprocal of resistance and is usually given in micromhos per unit length.

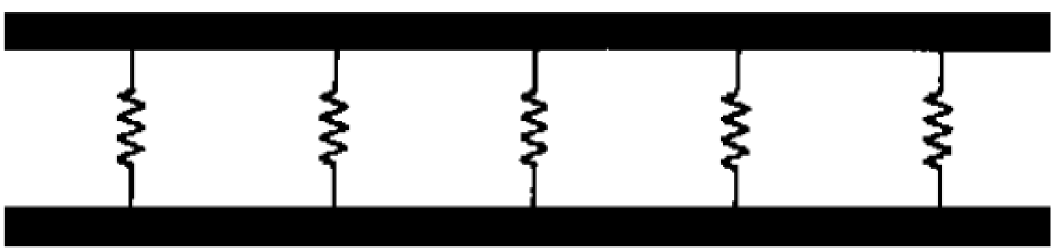


Figure 1-13.—Leakage in a transmission line.

CHARACTERISTIC IMPEDANCE OF A TRANSMISSION LINE

You learned earlier that the maximum (and most efficient) transfer of electrical energy takes place when the source impedance is matched to the load impedance. This fact is very important in the study of transmission lines and antennas. If the characteristic impedance of the transmission line and the load impedance are equal, energy from the transmitter will travel down the transmission line to the antenna with no power loss caused by reflection.

Definition and Symbols

Every transmission line possesses a certain CHARACTERISTIC IMPEDANCE, usually designated as Z_0. Z_0 is the ratio of E to I at every point along the line. If a load equal to the characteristic impedance is placed at the output end of any length of line, the same impedance will appear at the input terminals of the line. The characteristic impedance is the only value of impedance for any given type and size of line that acts in this way. The characteristic impedance determines the amount of current that can flow when a given voltage is applied to an infinitely long line. Characteristic impedance is comparable to the resistance that determines the amount of current that flows in a dc circuit.

In a previous discussion, lumped and distributed constants were explained. Figure 1-15, view A,shows the properties of resistance, inductance, capacitance, and conductance combined in a short section of two-wire transmission line. The illustration shows the evenly distributed capacitance as a single lumped capacitor and the distributed conductance as a lumped leakage path. Lumped values may be used for transmission line calculations if the physical length of the line is very short compared to the wavelength of energy being transmitted. Figure 3-15, view B, shows all four properties lumped together and represented by their conventional symbols.

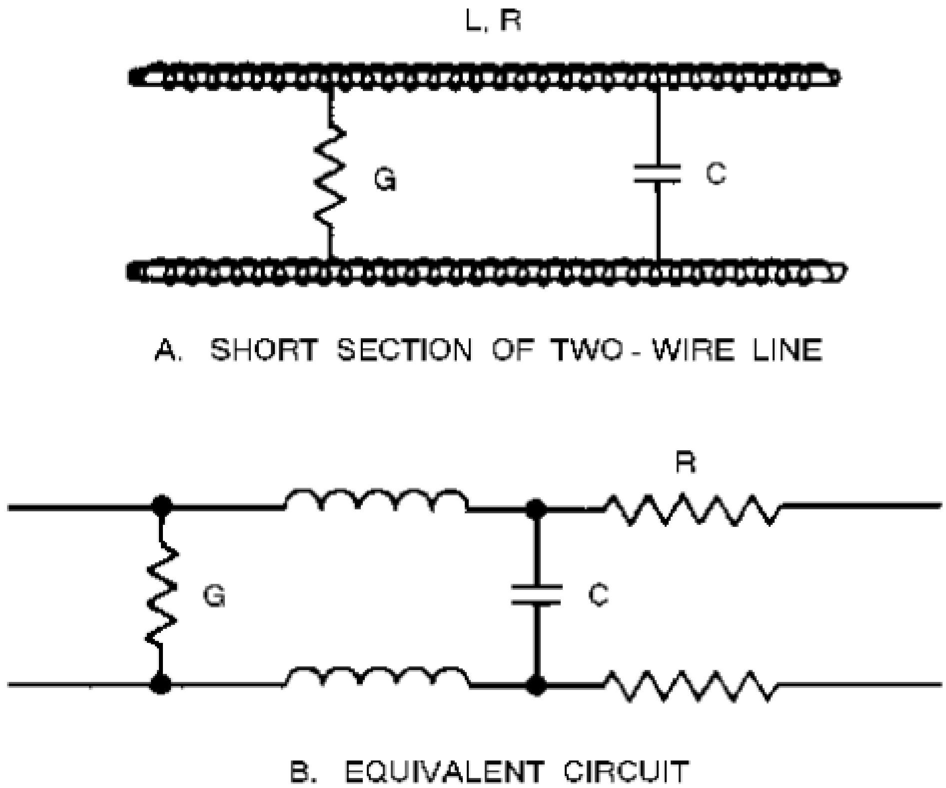


Figure 1-15 —Short section of two-wire transmission line and equivalent circuit.

VOLTAGE CHANGE ALONG A TRANSMISSION LINE

Let us summarize what we have just discussed. In an electric circuit, energy is stored in electric and magnetic fields. These fields must be brought to the load to transmit that energy. At the load, energy contained in the fields is converted to the desired form of energy.

Transmission of Energy

When the load is connected directly to the source of energy, or when the transmission line is short, problems concerning current and voltage can be solved by applying Ohm's law. When the transmission line becomes long enough so the time difference between a change occurring at the generator and the change appearing at the load becomes appreciable, analysis of the transmission line becomes important.

Transmission Line – A two conductor structure that can support a TEM wave.

TEM wave: An electromagnetic wave wherein both the electric and magnetic fields are perpendicular to the direction of wave propagation.

A passive, linear, two port device that allows bounded E. M. energy to flow from one device to another

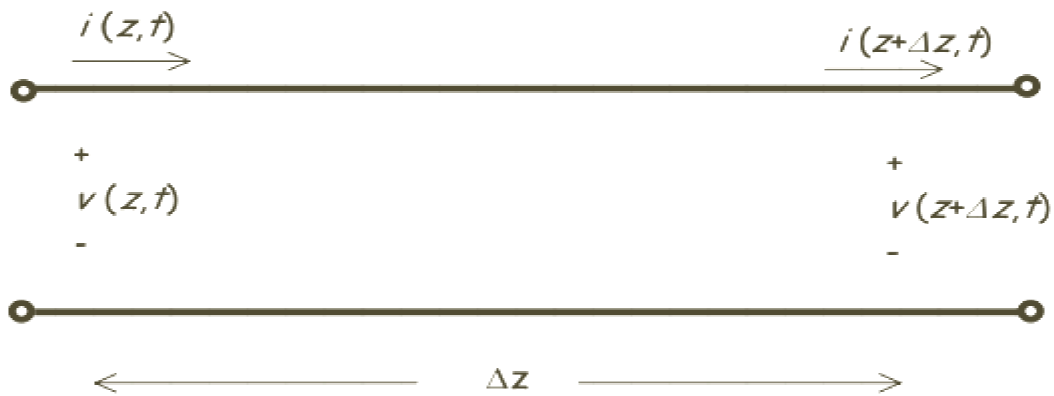


Figure 1-16.—Transmission line.

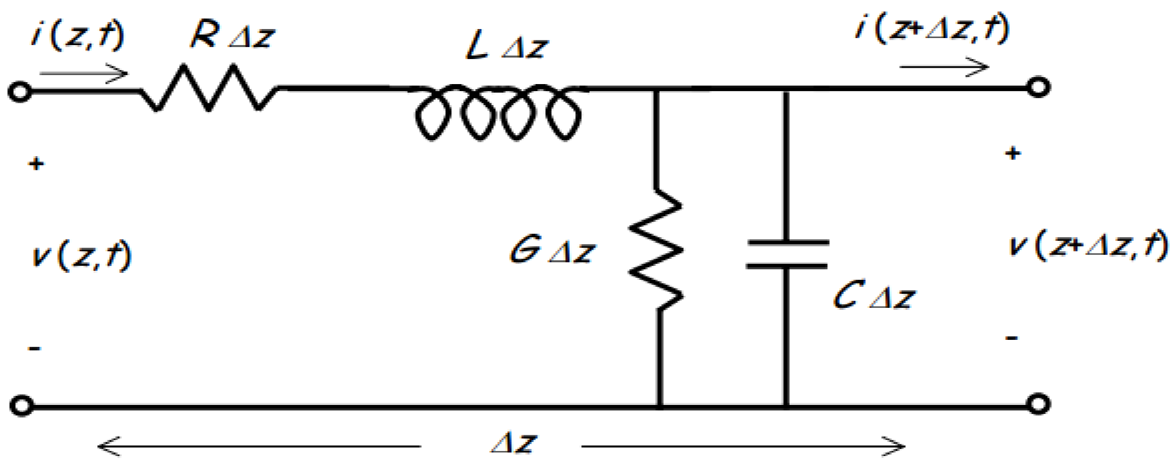


Figure 1-17.—Lumped-element equivalent circuit.

Where: R = resistance/unit length L = inductance/unit length C = capacitance/unit length G = conductance/unit length
∴ resistance of wire length Δz is RΔz.

Distributed elements

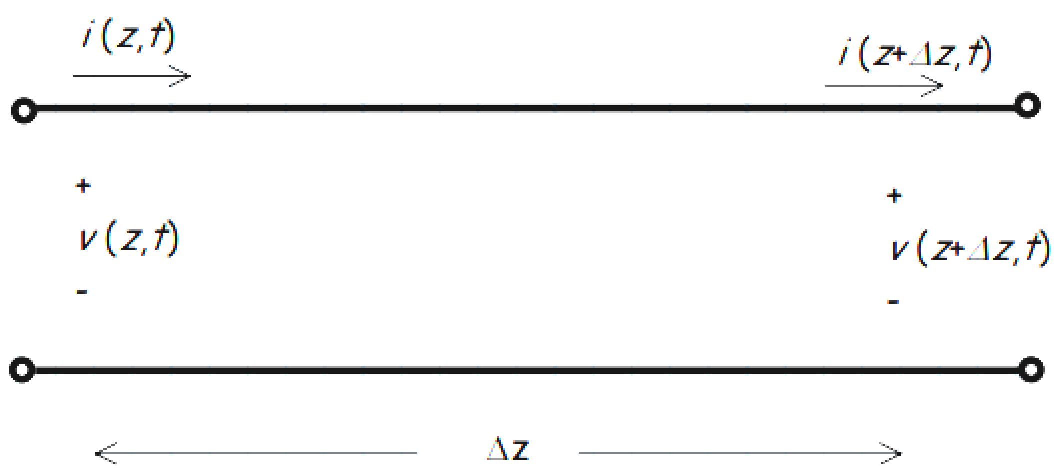
The line parameters R, L, C and G are distributed over the entire length of the transmission line. Hence they are called distributed parameters. They are also called primary constants.

Line parameters of a transmission line

The line parameters of a transmission line are resistance, inductance, capacitance and conductance.
Resistance (R) is defined as the loop resistance per unit length of the transmission line. Its unit is ohms/km.
Inductance (L) is defined as the loop inductance per unit length of the transmission line. Its unit is Henries/km.
Capacitance (C) is defined as the shunt capacitance per unit length between the two transmission lines. Its unit is Farad/km.
Conductance (G) is defined as the shunt conductance per unit length between the two transmission lines. Its unit is mhos/km.

The Telegrapher Equations

Consider a section of "wire":



i.e.,

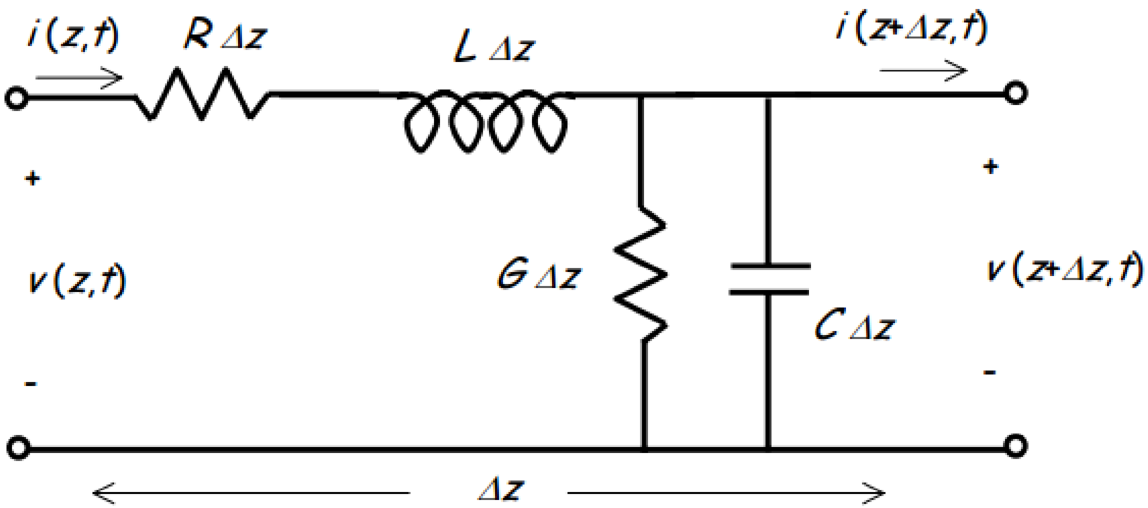


Figure 1-18.—Lumped-element equivalent circuit.

Where: R = resistance/unit length L = inductance/unit length C = capacitance/unit length G = conductance/unit length

∴ resistance of wire length Δz is RΔz.

Using KVL, we find:

$$v(z+\Delta z,t)-v(z,t)=-R\Delta z\, i(z,t)-L\Delta z\frac{\partial i(z,t)}{\partial t}$$

and from KCL:

$$i(z+\Delta z,t)-i(z,t)=-G\Delta z\, v(z,t)-C\Delta z\frac{\partial v(z,t)}{\partial t}$$

Dividing the first equation by Δz, and then taking the limit as $\Delta z \rightarrow 0$:

$$\lim_{\Delta z \rightarrow 0} \frac{v(z+\Delta z,t)-v(z,t)}{\Delta z} = -R\,i(z,t) - L\frac{\partial i(z,t)}{\partial t}$$

which, by definition of the derivative, becomes:

$$\frac{\partial v(z,t)}{\partial z} = -R\,i(z,t) - L\frac{\partial i(z,t)}{\partial t}$$

Similarly, the KCL equation becomes:

$$\frac{\partial i(z,t)}{\partial z} = -G\,v(z,t) - C\frac{\partial v(z,t)}{\partial t}$$

If $v(z,t)$ and $i(z,t)$ have the form:

$$v(z,t) = \mathrm{Re}\left\{V(z)e^{j\omega t}\right\} \quad \text{and} \quad i(z,t) = \mathrm{Re}\left\{I(z)e^{j\omega t}\right\}$$

then these equations become:

$$\frac{\partial V(z)}{\partial z} = -(R + j\omega L)\,I(z)$$

$$\frac{\partial I(z)}{\partial z} = -(G + j\omega C)\,V(z)$$

These equations are known as the **telegrapher's equations** !

- The functions I(z) and V(z) are complex, where the magnitude and phase of the complex functions describe the magnitude and phase of the sinusoidal time function $e^{j\omega t}$.

- Thus, I(z) and V(z) describe the current and voltage along the transmission line, as a function as position z.

- Remember, not just any function I(z) and V(z) can exist on a transmission line, but rather only those functions that satisfy the telegraphers equations.

The Transmission Line Wave Equation

We will combine the telegrapher equations to form one differential equation for V (z) and another for I (z).

First, take the derivative with respect to z of the first telegrapher equation:

$$\frac{\partial}{\partial z}\left\{\frac{\partial V(z)}{\partial z} = -(R + j\omega L)I(z)\right\}$$

$$= \frac{\partial^2 V(z)}{\partial z^2} = -(R + j\omega L)\frac{\partial I(z)}{\partial z}$$

Note that the second telegrapher equation expresses the derivative of I(z) in terms of V(z):

$$\frac{\partial I(z)}{\partial z} = -(G + j\omega C)\, V(z)$$

Combining these two equations, we get an equation involving V (z) only:

$$\frac{\partial^2 V(z)}{\partial z^2} = (R + j\omega L)(G + j\omega C)\, V(z)$$

$$= \gamma^2 V(z)$$

Where it is apparent that:

$$\gamma^2 \doteq (R + j\omega L)(G + j\omega C)$$

In a similar manner (i.e., begin by taking the derivative of the second telegrapher equation), we can derive the differential equation:

$$\frac{\partial^2 I(z)}{\partial z} = \gamma^2 I(z)$$

We have decoupled the telegrapher's equations, such that we now have two equations involving one function only:

$$\frac{\partial^2 V(z)}{\partial z} = \gamma^2 V(z)$$

$$\frac{\partial^2 I(z)}{\partial z} = \gamma^2 I(z)$$

Note only special functions satisfy these equations: if we take the double derivative of the function, the result is the original function (to within a constant)!

Therefore, the general solution to these wave equations (and thus the telegrapher equations) are:

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}$$

where V_0^+ ,V_0^-, I_0^+ , I_0^-, and γ are complex constants.

It means that the functions V(z) and I(z), describing the current and voltage at all points z along a transmission line, can always be completely specified with just four complex constants (V_0^+ ,V_0^-, I_0^+ , I_0^-).

We can alternatively write these solutions as:

$$V(z) = V^{+}(z) + V^{-}(z)$$

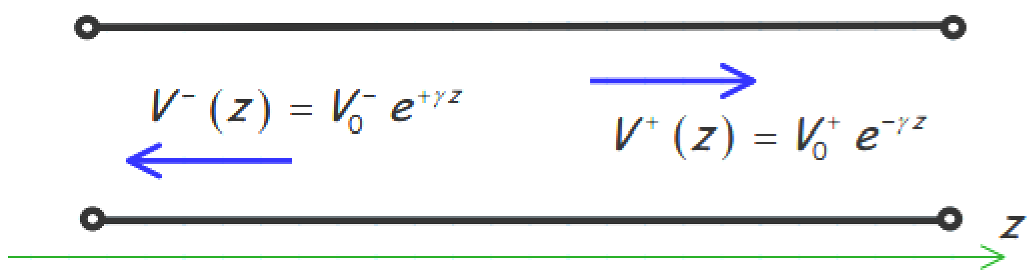
$$I(z) = I^{+}(z) + I^{-}(z)$$

where:

$$V^{+}(z) \doteq V_0^{+} e^{-\gamma z} \qquad V^{-}(z) \doteq V_0^{-} e^{+\gamma z}$$

$$I^{+}(z) \doteq I_0^{+} e^{-\gamma z} \qquad I^{-}(z) \doteq I_0^{-} e^{+\gamma z}$$

The two terms in each solution describe two waves propagating in the transmission line, one wave ($V^{+}(z)$ or $I^{+}(z)$) propagating in one direction (+z) and the other wave ($V^{-}(z)$ or $I^{-}(z)$) propagating in the opposite direction (-z).



Therefore, we call the differential equations introduced in this learning module the transmission line wave equations.

The Characteristic Impedance of a Transmission Line

So, from the telegrapher's differential equations, we know that the complex current I(z) and voltage V (z) must have the form:

$$V(z) = V_0^{+} e^{-\gamma z} + V_0^{-} e^{+\gamma z}$$

$$I(z) = I_0^{+} e^{-\gamma z} + I_0^{-} e^{+\gamma z}$$

Let's insert the expression for V (z) into the first telegrapher's equation, and see what happens!

$$\frac{dV(z)}{dz} = -\gamma V_0^+ e^{-\gamma z} + \gamma V_0^- e^{+\gamma z} = -(R + j\omega L)I(z)$$

Therefore, rearranging, I (z) must be:

$$I(z) = \frac{\gamma}{R + j\omega L}(V_0^+ e^{-\gamma z} - V_0^- e^{+\gamma z})$$

$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z} = \frac{\gamma}{R + j\omega L}(V_0^+ e^{-\gamma z} - V_0^- e^{+\gamma z})$$

For the above equation to be true for all z, I_0 and V_0 must be related as:

$$I_0^+ e^{-\gamma z} = \left(\frac{\gamma}{R + j\omega L}\right) V_0^+ e^{-\gamma z} \quad \text{and} \quad I_0^- e^{+\gamma z} = \left(\frac{-\gamma}{R + j\omega L}\right) V_0^- e^{+\gamma z}$$

Or—recalling that $V_0^+ e^{-\gamma z} = V^+(z)$ (etc.)—we can express this in terms of the two propagating waves:

$$I^+(z) = \left(\frac{\gamma}{R + j\omega L}\right) V^+(z) \quad \text{and} \quad I^-(z) = \left(\frac{-\gamma}{R + j\omega L}\right) V^-(z)$$

Now, we note that since:

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

We find that:

$$\frac{\gamma}{R + j\omega L} = \frac{\sqrt{(R + j\omega L)(G + j\omega C)}}{R + j\omega L} = \sqrt{\frac{G + j\omega C}{R + j\omega L}}$$

Thus, we come to the startling conclusion that:

$$\frac{V^+(z)}{I^+(z)} = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \quad \text{and} \quad \frac{-V^-(z)}{I^-(z)} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

Note that although the magnitude and phase of each propagating wave is a function of transmission line position z (e.g., $V^+(z)$ and $I^+(z)$), the ratio of the voltage and current of each wave is independent of position—a constant with respect to position z.

Although $V_0^{\pm}$ and $I_0^{\pm}$ are determined by boundary conditions (i.e., what's connected to either end of the transmission line), the ratio $V_0^{\pm}\ I_0^{\pm}$ is determined by the parameters of the transmission line only (R, L, G, C).

This ratio is an important characteristic of a transmission line, called its Characteristic Impedance Z_0.

$$Z_0 \doteq \frac{V_0^+}{I_0^+} = \frac{-V_0^-}{I_0^-} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

We can therefore describe the current and voltage along a transmission line as:

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{+\gamma z}$$

or equivalently:

$$V(z) = Z_0 I_0^+ e^{-\gamma z} - Z_0 I_0^- e^{+\gamma z}$$

$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}$$

Note that instead of characterizing a transmission line with real parameters R, G, L, and C, we can (and typically do!) describe a transmission line using complex parameters Z_0 and γ .

The Complex Propagation Constant (γ)

Recall that the current and voltage along a transmission line have the form:

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{+\gamma z}$$

where Z_0 and γ are complex constants that describe the properties of a transmission line. Since γ is complex, we can consider both its real and imaginary components

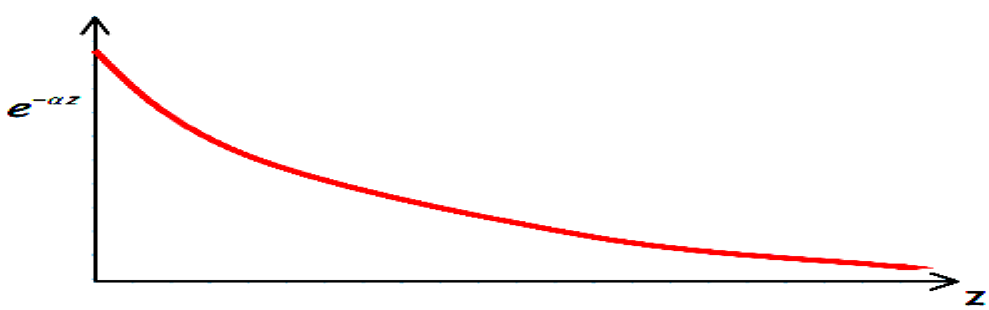
$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$
$$\doteq \alpha + j\beta$$

Where $\alpha =$ Re $\{\gamma\}$ and $\beta =$ Im $\{\gamma\}$. Therefore, we can write:

$$e^{-\gamma z} = e^{-(\alpha + j\beta) z} = e^{-\alpha z} e^{-j\beta z}$$

Since $e^{-j\beta z} = 1$, then $e^{-\alpha z}$ alone determines the magnitude of $e^{-\gamma z}$.

I.E., $|e^{-\gamma z}| = e^{-\alpha z}$.



Therefore, α expresses the attenuation of the signal due to the loss in the transmission line.

Since $e^{-\alpha z}$ is a real function, it expresses the magnitude of $e^{-\gamma z}$ only. The relative phase φ (z) of $e^{-\gamma z}$ is therefore determined by $e^{-j\beta z} = e^{-j\varphi z}$ only (recall $e^{-j\beta z} = 1$).

From Euler's equation:

$$e^{j\phi(z)} = e^{j\beta z} = \cos(\beta z) + j\sin(\beta z)$$

Therefore, βz represents the relative phase φ (z) of the oscillating signal, as a function of transmission line position z. Since phase φ(z) is expressed in radians, and z is distance (in meters), the value β must have units of :

β= φ/ z radians/ meter

The wavelength λ of the signal is the distance $z\Delta_{2\pi}$ over which the relative phase changes by 2π radians. So:

$$2\pi = \phi(z + \Delta z_{2\pi}) - \phi(z) = \beta\,\Delta z_{2\pi} = \beta\lambda$$

or, rearranging:

$$\beta = \frac{2\pi}{\lambda}$$

Since the signal is oscillating in time at rate ω rad /sec, the propagation velocity of the wave is:

$$v_p = \frac{\omega}{\beta} = \frac{\omega\lambda}{2\pi} = f\lambda \qquad \left(\frac{m}{sec} = \frac{rad}{sec}\,\frac{m}{rad}\right)$$

where f is frequency in cycles/sec.
Recall we originally considered the transmission line current and voltage as a function of time and position (i.e. (z, t) and i (z, t)). We assumed the time function was sinusoidal, oscillating with frequency ω:

$$v(z,t) = Re\{V(z)e^{j\omega t}\}$$

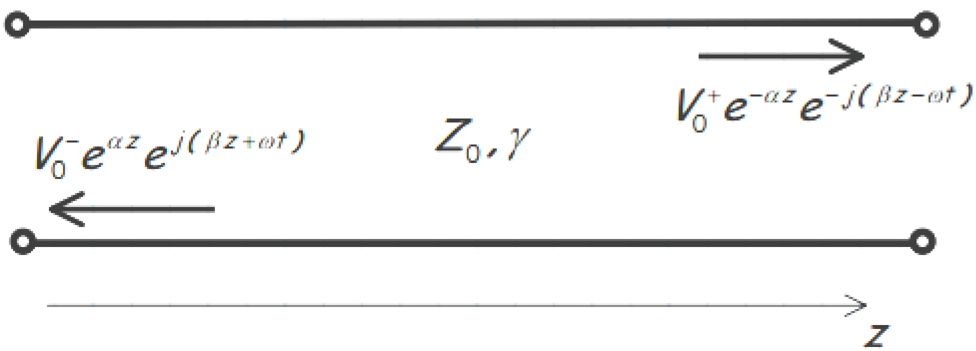
$$i(z,t) = Re\{I(z)e^{j\omega t}\}$$

Now that we know V(z) and I(z), we can write the original functions as:

$$v(z,t) = Re\left\{V_0^+ e^{-\alpha z} e^{-j(\beta z-\omega t)} + V_0^- e^{\alpha z} e^{j(\beta z+\omega t)}\right\}$$

$$i(z,t) = Re\left\{\frac{V_0^+}{Z_0} e^{-\alpha z} e^{-j(\beta z-\omega t)} - \frac{V_0^+}{Z_0} e^{\alpha z} e^{j(\beta z+\omega t)}\right\}$$

The first term in each equation describes a wave propagating in the +z direction, while the second describes a wave propagating in the opposite (-z) direction.



Each wave has wavelength:

$$\lambda = \frac{2\pi}{\beta}$$

And velocity:

$$v_p = \frac{\omega}{\beta}$$

Phase and group velocity of waves

To understand the difference between phase and group velocity of waves, consider the following analogy. A group of people, say village gudlavalleru runners, start from the starting at the same time. Initially it would appear that all of them are running at the same speed. As time passes, group spreads out (disperses) simply because each runner in the group is running with different speed. If you think of **phase velocity to be like the speed of an individual runner**, then the **group velocity is the speed of the entire group as a whole**. Obviously and most often, individual runners can run faster than the group as a whole. To stretch this analogy, we note that the phase velocity v_p of waves are typically larger than the group velocity v_g of waves. However, this really depends on the properties of the medium. The media in which $v_g = v_p$ is called the non-dispersive medium. But the media in which $v_g < v_p$ is called normal dispersion. The media in which $v_g > v_p$ is called anomalous dispersive media. It must be emphasized that dispersion is a property of the medium in which a wave travels. It is not the property of the waves themselves. The relation between phase and group velocity is given by, $v_g = d\omega/dk = v_p - \lambda\, dv_p / d\lambda$ Generally, $\omega(k)$ is called the dispersion relation and indicates the dispersion properties of a medium. As this formula predicts, if the phase velocity does not depend on the wavelength of the propagating wave, then $v_g = v_p$.

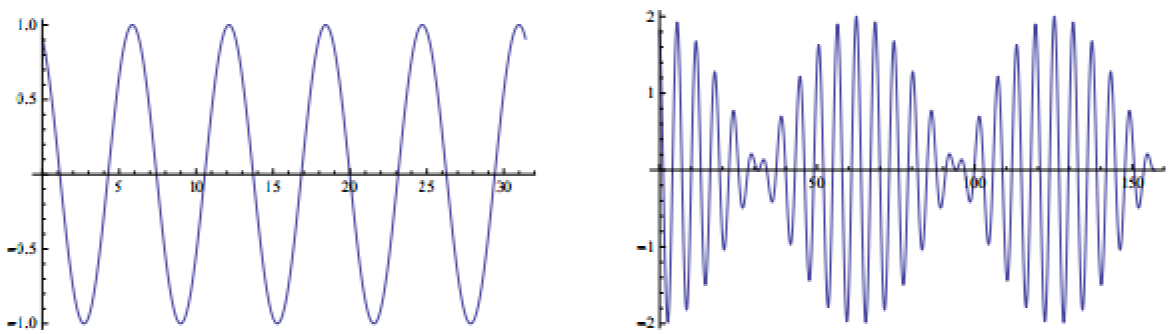


Figure 1.19: (left) A single travelling wave with frequency $\omega = 1$. (right) A group of waves composed of two waves with frequencies $\omega = 1$ and $\omega = 1.1$.

Briefly phase velocity refers to the velocity of a monochromatic wave, let's say, the velocity of one of the peaks of the wave. For example, a monochromatic wave with angular frequency $\omega = 2\pi\nu$ (ν is the frequency) travelling in +ve x-direction is given by, $y = A \sin(\omega t - kx)$. On the other hand, group velocity refers to a group composed of waves within a frequency band $\Delta\omega$. Group velocity is the velocity with which the entire group of waves would travel. The following figure 1.19 shows $y = \sin(2 + t)$ and $y = \sin(2 + t) + \sin(2 + 1.1t)$. The last form is the sum of two waves whose frequencies differ by 0.1. Notice that the amplitude of the group is modulated as a function of t. The example here shows the waves as a function of t, but similar scenario holds good for waves as a function of x. For the travelling wave shown in the left panel of the figure, phase velocity is the velocity with which any one of the peaks progresses. However, for the right panel of the figure, the speed of any of the peaks would give the group velocity.

Assignment-Cum-Tutorial Questions

A. Questions testing the remembering / understanding level of students

I) Objective Questions

1. A transmission line can be represented as

   a) a circuit which contains R & L in series and G & C in shunt.

   b) a circuit which contains R & G in series and L & C in shunt.

   c) a circuit which contains R & C in series and G & L in shunt.

   d) none of these.
2. Give the shunt admittance of Transmission line.
3. Draw the approximate equivalent circuit of length Δx of a transmission line.
4. Primary constants of transmission line are _______________.
5. Secondary constants of transmission line are _______________.
6. Series Impedance of Transmission line is given by---------------
7. What is the input impedance of Infinite length Transmission line?
8. What is meant by Phase velocity of Transmission line?
9. What is the input impedance of transmission line terminated with its Characteristic Impedance?
10. Give the relation between series impedance, shunt admittance and Characteristic impedance of Transmission line?

II) Descriptive Questions

1. What are the different types of Transmission lines? Explain.
2. Derive the Transmission line Equations.
3. Derive the input impedance and transmission line equations of Infinite length Transmission line.
4. Show that the input impedance of transmission line terminated with Characteristic impedance is equal to characteristic impedance.
5. Explain (i) Characteristic impedance (ii) Propagation Constant of Transmission line
6. Explain Wavelength, Phase Velocity and Group velocity of transmission line

B. Question testing the ability of students in applying the concepts.

I) 1. A generator of 1 volt,1000Hz,supplies power to 1000km long open wire line terminated in Z_o (characteristic impedance) and having following parameters:

R=10.4 ohms, L=0.0037 Henry, G=0.8 micromhos, C=0.00835 mfd.

Calculate the Phase velocity, Characteristic impedance, Propagation constant.

2. The primary constants of a line per loop Km are R=196 ohms, C=0.09 mfd, L=0.71Mh and leakage conductance negligible. Calculate the characteristic impedance and the propagation constant of 5.00/2π Hz.

3. A transmission line has the following constants

R=10.4 ohms, L=3.66 mH, C=0.00835 mfd, G=0.08microhms.

Calculate Z_0, α, β and V_p at ω =5,000 radians per sec.

4. The parameters of the line are

R=65ohms/km, L=1.6 mH/km, C=0.1μF/Km, G=2.25μmho/Km

Calculate the characteristic impedance.

5. An open wire transmission line terminated in its characteristic impedance has the following primary constants

R=6 ohms/km L=2 mH/km G=0.5 μΩ/km C=0.005 μ/loop km

Calculate the phase velocity and the attenuation in dB suffered by a signal in length of 100km.

6. A parallel wire line is made up of two copper conductors each of radius 1 mm separated by a distance of 30 cm. in air. Conductivity of copper is $5.75X10^7$ mho/meter. Calculate the d.c. resistance, inductance and capacitance per kilometer of the line. Also calculate the a.c. resistance of the line at frequency of 30 kHz.

II)

1. The characteristic impedance of a certain line is $710\angle -16°$ when the frequency is 1KHz. At this frequency the attenuation is 0.01 neper per km and the phase function is 0.035 radians per km. calculate the resistance, the leakage, the inductance and the capacitance per km and velocity of propagation.
2. The characteristic impedance of a uniform transmission line is 2039.5 ohm at frequency of 800 HZ. At this frequency the propagation constant was found to be $0.054\angle 87.9^o$.Determine the values of line constants R, L, G and C.
3. An open wire telephone line has R= 10 ohm per km L=0.0037 henry per km, $C=0.0083*10^{-6}$ farad per km and $G=0.4*10^{-6}$ ohms per km. Determine its Z_o, α and β at 1000 HZ.
4. The constant of a L.F; transmission line per Km are R=6 ohms, L=2.2mH,c=0.005 μF , G=0.25micro mho. Calculate at the frequency of 1000Hz.(¡)the terminating impedance for which no reflection will be set up in the line.(ii)the attenuation in dB suffered by signal at 1000Hz, while travelling a distance of 100Km when the line is properly terminated and the phase velocity with which the signal would travel.
5. A telephone line has resistance of 20 ohms , inductance of 10 mH. Capacitance of 0.1μF , and insulation resistance of 0.1 mega ohm/km. Find the input impedance at angular frequency of 5000 radian/sec., if the line is very long.

6. A 12km line is terminated by its characteristic impedance. At a certain frequency the voltage at 1km from the sending end it 10% below at the sending end. Find the voltage across the load impedance in terms of percentage of the sending end voltage.

UNIT-I
TRANSMISSION LINES-I
Assignment-Cum-Tutorial Questions

SECTION-A

1. A transmission line can be represented as [ ]
   a) a circuit which contains R & L in series and G & C in shunt.
   b) a circuit which contains R & G in series and L & C in shunt.
   c) a circuit which contains R & C in series and G & L in shunt.
   d) none of these.

2. A practical transmission line has propagation constant equal to [ ]
   A) α- jβ B) $\alpha\beta$ C) α+jβ D) α/jβ
3. Primary constants of transmission line are _______________.
4. Secondary constants of transmission line are _______________.
5. Series Impedance of Transmission line is given by_____________.
6. Give the shunt admittance of Transmission line.
7. Draw the approximate equivalent circuit of length Δz of a transmission line.
8. What is the input impedance of Infinite length Transmission line?
9. What is meant by Phase velocity of Transmission line?
10. What is the input impedance of transmission line terminated with its Characteristic Impedance?
11. Give the relation between series impedance, shunt admittance and Characteristic impedance of Transmission line?
12. If the load impedance of a transmission line is 200 ohms and its characteristic impedance is 200 ohms, find out the input impedance?
13. Write the equations for solutions of transmission lines.
14. The parameters of the line are R=65ohms/km, L=1.6 mH/km, C=0.1μF/Km, G=2.25μmho/Km, Calculate the characteristic impedance.
15. The lossless transmission line satisfies the following condition(s) [ ]

    a) R = 0 b) G = 0 c) R = 0 and G = 0 d) $\frac{R}{L} = \frac{G}{C}$

SECTION-B

Descriptive Questions

1. What are the different types of Transmission lines? Explain.**[C01]**
2. Derive the Transmission line Equations. **[C01]**
3. Derive the input impedance and transmission line equations of Infinite length Transmission line. **[C02]**
4. Derive the solutions of transmission line equations terminated with any load impedance. **[C01]**
5. Show that the input impedance of transmission line terminated with Characteristic impedance is equal to characteristic impedance. **[C01]**
6. Explain (i) Characteristic impedance (ii) Propagation Constant of Transmission line. **[C01]**
7. Explain Wavelength, Phase Velocity and Group velocity of transmission line. **[C01]**

8. Derive the expression for input impedance of a transmission line terminated with any load impedance other than characteristic impedance. **[C02]**

Problems

1. The primary constants of a line per loop Km are R=196 ohms, C=0.09 mfd, L=0.71Mh and leakage conductance negligible. Calculate the characteristic impedance and the propagation constant of 5.00/2π Hz. **[C02]**

2. A transmission line has the following constants. R=10.4 ohms, L=3.66 mH, C=0.00835 mfd, G=0.08microhms. Calculate Z_0, α, β and V_p at ω =5,000 radians per sec. **[C02]**

3. The series inductance and shunt capacitance of a general transmission line are given as 2 mH/km and 0.005 μF/km respectively. However, series resistance and shunt conductance of this transmission line R = G = 0. Calculate the characteristic impedance. **[C01]**

4. The characteristic impedance of a uniform transmission line is 2039.5 ohm at frequency of 800 HZ. At this frequency the propagation constant was found to be $0.054\angle 87.9^{o}$.Determine the values of line constants R, L, G and C. **[C01]**

5. An open wire telephone line has R= 10 ohm per km L=0.0037 henry per km, C=0.0083*10^{-6} farad per km and G=0.4*10^{-6} ohms per km. Determine its Z_o, α and β at 1000 HZ. **[C02]**

6. At 8 MHz the characteristic impedance of a transmission line is 40-j2 ohms and the propagation constant is 0.01+j0.018 per meter. Find the primary constants. **[C01]**

7. A telephone line has resistance of 20 ohms , inductance of 10 mH. Capacitance of 0.1μF , and insulation resistance of 0.1 mega ohm/km. Find the input impedance at angular frequency of 5000 radian/sec., if the line is very long. **[C01]**

8. An open wire transmission line terminated in characteristic impedance has the following primary constants at 2 KHz, R=12 Ω/km, L= 4 mH/km, C=0.005 μf/km, G=1μmho/km. Calculate the phase velocity and the attenuation in dB suffered by a signal in a length of 200km.[1nep=8.686 dB] **[C02]**

SECTION-C

1. The following are the Secondary constants of a transmission line [ ]

   A) α, β, λ, f B) R, L, C, G C) γ, Z_0 D) μ_0, ε_0

2. Propagation constant of a transmission line = ________________ [ ]

   A) $\sqrt{(R+j\omega L)(G+j\omega C)}$ B) $\sqrt{\frac{(R+j\omega L)}{(G+j\omega C)}}$ C) $\sqrt{\frac{L}{C}}$ D) $\frac{\omega}{\beta}$

3. Phase velocity of a transmission line =______________________ [ ]

   A) $\sqrt{(R+j\omega L)(G+j\omega C)}$ B) $\sqrt{\frac{(R+j\omega L)}{(G+j\omega C)}}$ C) $\sqrt{\frac{L}{C}}$ D) $\frac{\omega}{\beta}$

4. Microstrip lines are used in [ ]

B) PCBs B) filters C) vacuum tubes D) none

Transmission lines and Waveguides(UNIT II)

A. Questions testing the remembrance/understanding level of students
I. Objective/Multiple choice questions

1. Eighth-wave line transforms any resistance to impedance with a magnitude equal to ------ of the line
2. ------ wave line acts as *impedance transformer* or *inverter*.
3. An open circuited line with length < λ/4 is equivalent to-------.
4. An open circuited λ/4 line is equivalent to------- circuit.
5. An loen circuited line with length > λ/4 is equivalent to an--------.
6. The impedance of a quarter wave line along its length is pure --------
7. The dependence of attenuation on frequency causes-------
8. Different phase delays of different components cause-------
9. To avoid delay distortion, the condition to be satisfied is -------
10. Distortion-less condition is ------
11. Loading is addition of------ to achieve ------ condition.
12. Load matching refers to termination of line with ------

II. Descriptive questions

1. Derive a relation for RC over the load.
2. Derive the inter relations between RC and line impedance
3. Derive the values of SWR for different types of terminations.
4. Derive an expression for the input impedance of a loss-less line which it is terminated by (a) a load $\mathbf{Z}_l$ (b) open (c) short circuit and draw the suitable sketches.

B. Questions testing the ability of students in applying the concepts
I. Multiple choice questions

1. The distortion-less line condition is
   a. a) $R/L = G/C$ b) $R/L > G/C$
   b. c) $R/L < G/C$ d) None of these
2. The loading of line refers to the connection of
   a. (a)Inductive coils b) Capacitive boxes
   b. (b)Both (a) and (b) d) None of these
3. The input impedance of quarter wave, $\lambda/4$ transformer is
   a. a) Terminal impedance b) Terminal admittance
   b. c) Characteristic impedance d) None of these
4. The input impedance of half-wave line i.e. $\lambda/2$ transformer is
   a. a) Terminal impedance b) terminal admittance
   b. c) Characteristic impedance d) None of these
5. The reflection coefficient right over the source is
   a. (a) $\dfrac{(\mathbf{Z}_s - \mathbf{Z}_o)}{(\mathbf{Z}_s + \mathbf{Z}_o)}$ (b) $\dfrac{(\mathbf{Z}_s + \mathbf{Z}_o)}{(\mathbf{Z}_s - \mathbf{Z}_o)}$
   b. (c) $\dfrac{(\mathbf{Z}_o - \mathbf{Z}_s)}{(\mathbf{Z}_o + \mathbf{Z}_s)}$ (d) None of these

6. The SWR is meaningful for
   a. a) Lossy lines b) loss-less lines
   b. c) Both (a) and (b) d) None of these
7. The SWR can be found from
   a. a) Magnitude of RC b) Phase of RC
   b. c) Both (a) and (b) d) None of these
8. The SWR is
   a. a) Constant of line and load b) varies over the line
   b. c) Both (a) and (b) d) None of these
9. The range of SWR is
   a. a) −1 to 1 through 0 b) +1 to infinity
   b. c) 0 to infinity d) None of these
10. The SWR for sc or oc terminated line is
    a. a) Zero b) one
    b. c) Infinity d) None of these

II. Problems

1. Determine the primary constants, *R*, *L*, *G*, and *C* for a distortion-less line working at 300MHz. Given that the line has characteristic impedance, Z_o =75Ω, attenuation constant, α=0.12Np/m, and wave velocity, $v=1.4\times10^8$m/s.
   ***Answers*:** 9.0 Ω/m, 5.356×10^{-7}H/m, 16×10^{-4} ℧/m, 95.22pF/m.
2. A loss-less 200Ω line is terminated on a load given by (200–*j*200)Ω. Given that the propagation constant is (0.040+*j*2.25)/m. Find reflection and transmission coefficients at load.
   ***Answers*:** 0.447∠–1.107, 1.26∠–0.32.
3. A loss-less 50Ω line is terminated on a load given by 100Ω. The magnitude of voltage in incident wave is 20V(rms). Determine SWR, maximum voltage and currents as well minimum voltage and currents over the line.
   ***Answers*:** 2, 37.71V, 754.24mA, 18.85V,377.12mA
4. A loss-less 75Ω line is terminated over a load with impedance (120+*j*80)Ω. (a)Find RC, **Γ** and SWR ρ. (b) Also work out how far from load the line impedance is pure real.
   ***Answers*:** (a) 0.435∠0.67, 2.54 (b) 0.053λ
5. A 150Ω loss-less line connects a signal of 1GHz to a load of 200Ω. The load power is 100mW. Evaluate (a) voltage RC, (b) VSWR (c) incident and reflected powers and (d) positions of $V_{\max}$, $I_{\max}$, $V_{\min}$ and $I_{\min}$.
   ***Answers*:** (a) 1/7 (b) 4/3 (c) 102.08mW, 2.08mW (d) $V_{\max}$ and $I_{\min}$ right over load, $V_{\min}$ and $I_{\max}$ at a distance of 7.5cm from load.
6. A loss-less 75Ω line, 5λ/8 in length, is terminated on a load $\mathbf{Z}_l$. Find out its input impedance $\mathbf{Z}_{in}$ when (a) $\mathbf{Z}_l$= *j*45Ω (b) $\mathbf{Z}_l$= 25–*j*65Ω.
   ***Answers*:** (a) *j*300Ω (b) (13.90+*j*2.87) Ω.
7. Determine the input impedance of a short circuited 50Ω coaxial line with β = 8.5rad/m when line length is (a)15cm(b)1.5m(c)3λ/4 and (d)λ/8.
   ***Answers*:** (a) *j*164.08Ω, (b) *j*9.29Ω, (c) *j*20.93kΩ, (d) *j*50Ω

C. Questions testing the Analyzing/evaluating/creative abilities of student

1. Differentiate SWR from RC.

2. Analyze and derive an expression for voltage and current of SWR over the line.

3. What is distortion-less condition? Derive the relation for distortion-less line condition on the primary constants.
4. What is loading ? Discuss different types of loading methods mentioning their relative merits and demerits.
5. What are properties and applications of eighth wave line, quarter wave line and half wave line? Given a list of their applications.

D. Previous GATE/IES questions

1. A transmission line with a characteristic impedance of 100Ω is used to match a 50 Ω section to a 200 Ω section. If the matching is to be done both at 429MHz and 1GHz, the length of the transmission line can be approximately(GATE2012)

(A) 82.5cm (B) 1.05m (C) 1.58m (D) 1.75m Ans C

2. A transmission line of characteristic impedance 50Ω is terminated in a load impedance $\mathbf{Z}_l$. The VSWR of the line is measured as 5 and the first of the voltage maxima in the line is observed at a distance of $\lambda/4$ from the load. The value of $\mathbf{Z}_l$ is (GATE2011)

(A) 10 (B) 250 (C) (19.23 + j46.15) (D) (19.23 – j46.15) Ans B

LINE DISTORTION

The deviation of the signal waveform at the output of the line from that at its input terminals is called *line distortion*. It is due to the fact that all frequencies in the waveform do not have same attenuation and same delay during the propagation. The characteristic impedance, attenuation and velocity of propagation on the line, by being functions of frequency are all causes of this deformation. The total deviation of the waveform from its originality is considered as sum of two components, namely, frequency distortion and delay distortion.

Frequency distortion is due to various frequency components of the signal undergoing different amounts of attenuation when the attenuation constant α is function of frequency. To eliminate this distortion the attenuation constant α must be made independent of frequency.

Phase or delay distortion is due to different frequency components of the signal undergoing different amounts of phase delays while reaching the destination, thus spoiling the original phase relation among them. To eliminate this, phase shift constant β must be made proportional to angular frequency ω.

Equalizers **:** Frequency distortion can be reduced by cascading lines with networks known as '*equalizers*'. Equalizer is a network whose attenuation versus frequency characteristic is just opposite to that of the line. Delay distortion can also be reduced with equalizers, but it must be designed in such a way that β for total circuit is proportional to ω. For audio transmission, only frequency distortion is serious problem whereas for video transmission both, frequency as well as phase distortions, cause severe trouble.

Distortion-less line **:** By definition, distortion-less line is one which transmits the input signal without any distortion. It can be found that a line becomes distortion-free when its primary constants are related by,

$$\frac{R}{L}=\frac{G}{C} \rightarrow CR = LG \qquad (15.1)$$

This mathematical condition for distortion-free transmission is known as Heaviside Condition as it was derived by Oliver Heaviside first time in 1887.

Proof: For the line to have neither frequency nor delay distortion, its attenuation constant and velocity of propagation should be independent of frequency.

As the propagation velocity is given by, $v=\omega/\beta$, for it to become frequency independent, the phase shift constant, available in Eq.(13.25), must be a direct function of frequency. It can happen only when the second radical is equal to ($RG+\omega^2LC$). Enforcing this condition, it can be obtained that,

$$\left(LG-CR\right)=0 \rightarrow CR = LG$$

Then, the phase shift constant becomes,

$$\beta=\sqrt{\frac{1}{2}\left[\left(\omega^2LC-RG\right)+\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}\right]}$$

$$=\omega\sqrt{LC} \qquad (15.2)$$

which is proportional to angular frequency, ω making the propagation velocity independent of frequency, thus eliminating delay distortion. The propagation velocity, for this case, becomes,

$$v=\frac{\omega}{\beta}=\frac{\omega}{\omega\sqrt{LC}}=\frac{1}{\sqrt{LC}} \qquad (15.3)$$

One can also be observed that, when the second radical is equal to ($RG+\omega^2LC$), the expression for attenuation constant, available in Eq.(13.24), becomes frequency independent. Enforcing this condition on the radical gives,

$$(LG - CR) = 0 \rightarrow CR = LG$$

With this incorporated, the attenuation constant becomes independent of the frequency, given by

$$\alpha = \sqrt{\frac{1}{2}\left[\left(RG - \omega^2 LC\right) + \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}\right]}$$
$$= \sqrt{RG} \qquad (15.4)$$

In addition to v, β and α, it is also instructive to consider the expression of characteristic impedance for distortion-less line. Incorporating Eq.(15.1) into it, results in,

$$\mathbf{Z}_o = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}} = \sqrt{\frac{L(R/L + j\omega)}{C(G/C + j\omega)}} = \sqrt{\frac{L}{C}} \qquad (15.5)$$

In Table 15.1, the relation between distortion-less line and loss-less line is summarized. Consider the products $\alpha\mathbf{Z}_o$, $\alpha/\mathbf{Z}_o$, $\mathbf{Z}_o/v$ and $1/\mathbf{Z}_o v$ giving R, G, L and C, respectively. These relations, thus, suggest a way by which one can determine the primary constants of a distortion-less line, when attenuation constant, characteristic impedance and phase velocity are given.

Loading: In actual lines, the primary constants are such that $R/L >> G/C$, because of lower values of G. To make the line distortion-less by enforcing Eq.(15.1), the usual practice is to decrease R/L instead of going for the increase of G/C for reasons, like increase in G leads to increase in losses and inefficient operation etc. The decrease in R/L is achieved, usually, by increasing the inductance, L instead of going for a decrease in R as it requires large conductors, large copper and hence, it is costly to go for a reduction in R. Increase in L is affected either by changing the line configuration or by connecting highly inductive coils to the line. The method of increasing the series inductance of line using inductive coils is called '*loading*'.

Table 1 Distortion-less line versus loss-less line.

S.No.	Properties	Distortion-less line	Loss-less line
1.	Primary constants	$(R/L) = (G/C)$	$R = G = 0$
2.	Characteristic impedance	$\mathbf{Z}_o = \sqrt{L/C}$	$\mathbf{Z}_o = \sqrt{L/C}$
3.	Attenuation constant	$\alpha = \sqrt{RG}$	$\alpha = 0$
4.	Phase shift constant	$\beta = \omega\sqrt{LC}$	$\beta = \omega\sqrt{LC}$
5.	Wave velocity over line	$v = 1/\sqrt{LC}$	$v = 1/\sqrt{LC}$

Loading coils are traditionally known as *Pupin coils* after Mihajlo Idvorski Pupin (1858 – 1935), a Serbian American physicist and physical chemist, and the process of inserting them is sometimes called *pupinization*. The concept of loading coils was a discovery of Oliver Heaviside in the 1860s. He found that added inductance was essential to avoid attenuation and time delay distortion of the transmitted signal.

Permalloy and Mu-metal are two alloys widely used in the design of loading coils. The first one is a magnetic nickel-iron annealed alloy with higher magnetic permeability is a discovery of Gustav Elmen in 1914. The second one, Mu-metal, invented in 1923 by a telegraph company in London, has magnetic properties similar to permalloy, but it has increased ductility with the addition of copper and allows the metal to be drawn into wire. Compared to permalloy, Mu-metal cable is easier to construct, and also its construction lends itself to a variable loading profile. Loading is of three types: lumped loading, continuous and patch loading and all are described briefly below.

Lumped loading: In this method, as shown in Figure 1(a), relativiely high inductance coils are introduced at definite and uniform intervals along the length of the line to increase its inductance. In earlier days, the loading coils used to be two windings on a iron dust or permalloy dust cores. Presently, however, molybdenum permalloy dust cores are being used, as they give high inductance with a rather small coil. It was Heaviside who first made the proposal, in 1893, of using discrete inductors at intervals along the line.

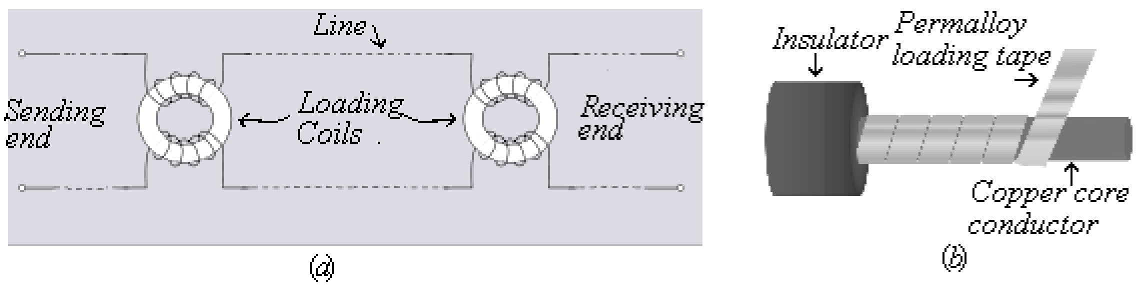


Figure 1 Loading methods. (a) Lumped loading and (b) continuous loading.

Continuous loading: The introduction of heavy loading coils in submarine telephone and telegraph cables can subject them to undue strain at the points of insertion. To avoid this issue, continuous loading, as shown in Figure 1(b), is used. The tape of steel or some other magnetic materials, such as 'perm alloy' or 'mumetal' is wrapped around the conductor to be loaded. As such loading distributes the inductance continuously along the line, it causes the line to behave like one with distributed constant. It increases the permeability, μ of the surrounding medium and thereby increasing the inductance. It is costly, and hence, used in sub-marine cables only.

Patch loading: Lumped loading is cheaper but it suffers with the drawbacks of a definite cutoff frequency and difficult seals. Continuous loading, on the other hand, is expensive and hence can be done only when it is absolutely necessary. A compromise method is patch loading, whereby the cable is continuously loaded in repeated sections, leaving the intervening sections unloaded.

Negative side of loading coils is that they cause distortion to higher frequencies, associated with digital signals, and hence, their presence in the line is not conducive for high speed data transmission. They are, however, highly useful to boost analog voice frequencies and are usually placed in local loops longer than 18,000 ft. In the current era, coils are hardly being used as they are superseded by higher technologies.

.

REFLECTION COEFFICIENT

To define RC, consider a line connected to a sinusoidal source and terminated over an arbitrary impedance. In the steady state, it can be found the existence of two waves over the line: one travelling towards the load, incident wave and the other towards the source, reflected wave. Each one these two waves is associated with voltage as well current. In general, the magnitude of the incident wave depends upon the source-line matching at the input end and that of the reflected wave depends upon the load-line matching at the termination end.

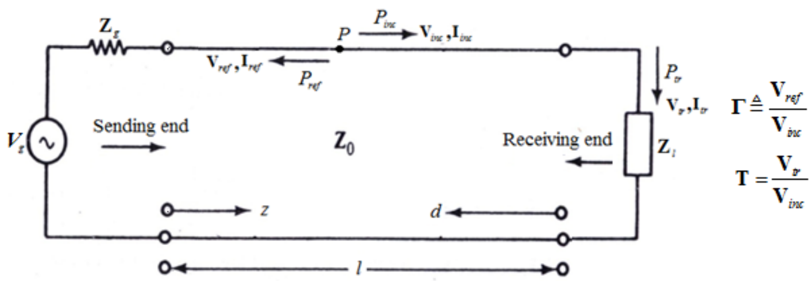


Figure 2 Defining Reflection Coefficient and Transmission Coefficient over a line.

With respect to this line, the vector ratio of voltage of in the reflected wave, $\mathbf{V}_{ref}$ to that in the incident wave, $\mathbf{V}_{inc}$ at an arbitrary point over the line is defined as *Reflection Coefficient* (RC) at that point and is usually denoted by $\mathbf{\Gamma}$ (Gamma). Mathematically,

$$\mathbf{\Gamma} \triangleq \frac{\mathbf{V}_{ref}}{\mathbf{V}_{inc}} \quad \rightarrow \quad \Gamma = |\mathbf{\Gamma}| = \frac{|\mathbf{V}_{ref}|}{|\mathbf{V}_{inc}|} \quad \& \quad \angle\mathbf{\Gamma} = \angle\mathbf{V}_{ref} - \angle\mathbf{V}_{inc}$$

As the voltages $\mathbf{V}_{ref}$ and $\mathbf{V}_{inc}$ are phasors, they are complex, and hence, their ratio, the RC is a complex quantity, denoted by bold face Greek letter, $\mathbf{\Gamma}$ and it′s magnitude by light face letter i.e., $\Gamma = |\mathbf{\Gamma}|$.

It can be easily shown that the ratio of currents in reflected and incident waves is equal to the *negative* of ratio of voltages of those waves. Accordingly, one can define the RC in terms of currents ratio also: the *negative* of ratio of current of the reflected wave to that of the incident wave. Mathematically,

$$\mathbf{\Gamma} \triangleq \frac{\mathbf{V}_{ref}}{\mathbf{V}_{inc}} = -\frac{\mathbf{I}_{ref}}{\mathbf{I}_{inc}}$$

For any passive line, the voltage amplitude in the reflected wave can never be more than that of the incidence wave, reflected wave is part of incident wave. Accordingly, the magnitude of the RC can never be more than one, always in between 0 and 1. In general, RC varies from point to point over the line and its range is from '–1' to '1' through '0'. In case of loss-less lines, however, the magnitude of RC remains same over the entire line. For this type of lines, as the magnitudes of the voltages, $\mathbf{V}_{ref}$ and $\mathbf{V}_{inc}$ remain constant, the magnitude of RC which is their ratio, also remains same at all points over the line.

The magnitude of the RC *over the load* depends upon the terminating impedance. When the load is matched i.e. load impedance is equal to characteristic impedance, no reflected wave exists, and hence, the magnitude of RC becomes zero. For non-dissipative loads, like open circuit, short circuit and pure reactances, the incident wave gets reflected completely, and hence, it becomes equal to one. For other types of loads, the magnitude lies in between 0 and 1.

The phase of the RC is the difference of phases of reflected wave and incident waves. As these phases vary form point to point over the line, the phase of RC also changes accordingly, as shown in the Figure 14.1. Consider the location of $\mathbf{V}_{max}$. Maximum occurs when reflected and incident waves are in phase, leading to phase of RC there as zero, and hence, it may be considered as phase reference. When points towards source are considered, the phase is negative and keep on increasing, reaching $-\pi$ at next minimum, -2π at next maximum, -3π at next minimum and so on so forth. When points towards load are considered, the phase is positive and keep on increasing, reaching π at next minimum, 2π at next maximum, 3π at next minimum and so on so forth.

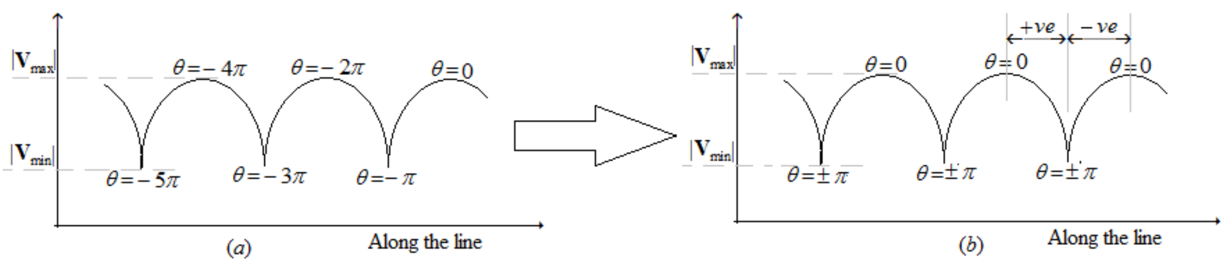


Figure 3 Phase of RC, θ at $\mathbf{V}_{max}$ and $\mathbf{V}_{min}$ over loss-less line. (a) Actual phase and (b) effective phase.

Now, if a $\mathbf{V}_{max}$ is considered, there the phase is 0, left side it is negative falling to $-\pi$ at the next $\mathbf{V}_{min}$, right side it is positive increasing to π at the next $\mathbf{V}_{min}$. Now, if a $\mathbf{V}_{min}$ is considered, there the phase is $\pm\pi$, left side it is positive decreasing to 0 at the next $\mathbf{V}_{max}$, right side it is negative increasing to 0 at the next $\mathbf{V}_{max}$.

.

In the standing wave pattern, the coefficient assumes pure real values at maxima and minima points. It is positive at voltage maxima(or current minima) and negative at minima (or current maxima) points of the pattern. Maxima occur due to constructive interference at points where reflected and incident waves differ in phase by $-2(k-1)\pi$ and minima occur due to destructive interference at points where reflected and incident waves differ in phase by $-(2k-1)\pi$, here k is integer i.e., k =1,2,3,.. etc. In either case, from the definition, one can see that the RC is pure real there.

Another parameter related to RC is transmission coefficient. When line is terminated on a mismatched load, the power incident over the load goes into it only partly, the remaining being reflected back. Transmission coefficient denotes the power flow into the load. By definition it is given by

$$\mathbf{T} = \frac{\text{Transmitted voltage or current}}{\text{Incident voltage or current}} = \frac{\mathbf{V}_{tr}}{\mathbf{V}_{inc}} = \frac{\mathbf{I}_{tr}}{\mathbf{I}_{inc}}$$

The transmission and reflection coefficients over the load are related through,

$$\mathbf{T} = 1 + \mathbf{\Gamma}_l$$

The incident wave plus reflected wave gives the transmitted wave at the load. Hence,

$$\mathbf{V}^{+}e^{-\gamma l} + \mathbf{V}^{-}e^{\gamma l} = \mathbf{V}_{tr}e^{-\gamma l} \rightarrow \frac{\mathbf{V}_{tr}e^{-\gamma l}}{\mathbf{V}^{-}e^{\gamma l}} = 1 + \frac{\mathbf{V}^{+}e^{-\gamma l}}{\mathbf{V}^{-}e^{\gamma l}} \rightarrow \mathbf{T} = 1 + \mathbf{\Gamma}_l$$

The reflection coefficient over load, $\mathbf{\Gamma}_l$ is related to load impedance, and hence, transmission coefficient also can be expressed in terms of load impedance.

$$\mathbf{\Gamma}_l = \frac{\mathbf{Z}_l - \mathbf{Z}_o}{\mathbf{Z}_l + \mathbf{Z}_o} \rightarrow \mathbf{T} = 1 + \mathbf{\Gamma}_l = \frac{2\mathbf{Z}_l}{\mathbf{Z}_l + \mathbf{Z}_o}$$

The connecting relation between transmission and RCs is,

$$\mathbf{T}^2 = \frac{\mathbf{Z}_l}{\mathbf{Z}_o}\left(1 - \mathbf{\Gamma}_l^2\right)$$

The powers carried by incident, reflected and transmitted waves, respectively, are,

$$\frac{(\mathbf{V}^+ e^{-\gamma l})^2}{2\mathbf{Z}_o}, \quad \frac{(\mathbf{V}^- e^{\gamma l})^2}{2\mathbf{Z}_o} \quad \& \quad \frac{(\mathbf{V}_{tr} e^{\gamma l})^2}{2\mathbf{Z}_l}$$

The difference of incident and reflected powers, naturally is transmitted power. Hence,

$$\frac{(\mathbf{V}^+ e^{-\gamma l})^2}{2\mathbf{Z}_o} - \frac{(\mathbf{V}^- e^{\gamma l})^2}{2\mathbf{Z}_o} = \frac{(\mathbf{V}_{tr} e^{\gamma l})^2}{2\mathbf{Z}_l} \rightarrow \mathbf{T}^2 = \frac{\mathbf{Z}_l}{\mathbf{Z}_o}\left(1 - \mathbf{\Gamma}_l^2\right)$$

In the above two relations, notice that transmission coefficient becoming 1 when load is matched i.e. $\mathbf{Z}_l = \mathbf{Z}_o$, indicating complete transmission into load. The RC at an arbitrary point over a uniform line can be expressed in terms of receiving end impedance, $\mathbf{Z}_r$ or sending end impedance, $\mathbf{Z}_s$. The procedures for the derivation of these relations are given below.

Example Find voltage and current RCs and also characteristic impedance of a uniform line having incident wave voltage and currents as 50∠0.50V and 0.667∠0.35A and those on reflected wave as 10∠0.65V and 0.133∠−2.64A

Also mention whether the line is loss-less or lossy.

Solution:

Voltage RC is,

$$\mathbf{\Gamma} = \frac{\mathbf{V}_{ref}}{\mathbf{V}_{inc}} = \frac{10\angle 0.65}{50\angle 0.50} = 0.2\angle 0.15 = \frac{0.133\angle -2.64}{0.667\angle 0.35} = -\frac{\mathbf{I}_{ref}}{\mathbf{I}_{inc}}$$

Current RC is found as, −0.2∠0.15V. It is verified that current RC is ratio of current in reflected wave to current in incident wave, also negative of voltage RC. The characteristic impedance is,

$$\mathbf{Z}_o = \frac{\mathbf{V}_{inc}}{\mathbf{I}_{inc}} = \frac{50\angle 0.50}{0.667\angle 0.35} = 75\angle 15\ \Omega = -\frac{\mathbf{V}_{ref}}{\mathbf{I}_{ref}}$$

Characteristic impedance is found as 75∠15V. It is verified that characteristic impedance is equal to negative of ratio of voltage to current ratio in the reflected wave.

Example A 75Ω loss-less line has a voltage of 60∠0.2V in its forward wave. The RC is found as1∠ 0.3 at that point. Find voltage and currents of the total wave.

Solution:

Voltage in the reverse wave is

$$\mathbf{V}_{ref} = \mathbf{\Gamma}\mathbf{V}_{inc} = 1\angle 0.3 \times 60\angle 0.2 = 60\angle 0.5\text{V}$$

Current in the forward wave is

$$\mathbf{I}_{inc} = \frac{\mathbf{V}_{inc}}{\mathbf{Z}_o} = \frac{60\angle 0.2}{75\angle 0} = 0.8\angle 0.2\text{A}$$

Current in the reverse wave is

$\mathbf{I}_{ref} = -\mathbf{\Gamma I}_{inc} = -1\angle 0.3 \times 0.8\angle 0.2 = 0.8\angle -2.642\text{A}$

Total voltage is sum of voltages in incident and reflected waves, and hence,

$\mathbf{V} = \mathbf{V}_{inc} + \mathbf{V}_{ref} = 60\angle 0.2 + 60\angle 0.5 = 118.65\angle 0.35\text{V}$

Total current is sum of currents in incident and reflected waves, and hence,

$\mathbf{I} = \mathbf{I}_{inc} + \mathbf{I}_{ref} = 0.8\angle 0.2 + 0.8\angle -2.642 = 0.24\angle -1.22\text{A}$

Example A (50+*j*0.2)Ω line is terminated on a load of impedance (a) (75+*j*50)Ω and (b) (75–*j*50)Ω. Determine RC and transmission coefficient over load.

Solution:

(a) $\mathbf{Z}_l$=(75+*j*50)Ω:: The RC and transmission coefficients over load are

$$\mathbf{\Gamma}_l = \frac{\mathbf{Z}_l - \mathbf{Z}_o}{\mathbf{Z}_l + \mathbf{Z}_o} = \frac{75 + j50 - 50 - j0.2}{75 + j50 + 50 + j0.2} = 0.414\angle 0.72$$

$$\mathbf{T} = \frac{2\mathbf{Z}_l}{\mathbf{Z}_l + \mathbf{Z}_o} = \frac{2(75 + j50)}{75 + j50 + 50 + j0.2} = 1.34\angle 0.206$$

As cross check,

$$1 + \mathbf{\Gamma}_l = 1 + 0.414\angle 0.72 = 1.34\angle 0.206 = \mathbf{T}$$

(b) $\mathbf{Z}_l$= (75–*j*50)Ω :: The RC and transmission coefficients over load are

$$\mathbf{\Gamma}_l = \frac{\mathbf{Z}_l - \mathbf{Z}_o}{\mathbf{Z}_l + \mathbf{Z}_o} = \frac{75 - j50 - 50 - j0.2}{75 - j50 + 50 + j0.2} = 0.417\angle -0.73$$

$$\mathbf{T} = \frac{2\mathbf{Z}_l}{\mathbf{Z}_l + \mathbf{Z}_o} \frac{2(75 - j50)}{75 - j50 + 50 + j0.2} = 1.34\angle -0.21$$

As cross check,

$$1 + \mathbf{\Gamma}_l = 1 + 0.417\angle -0.73 = 1.34\angle -0.21 = \mathbf{T}$$

Reflection Coefficient-Receiving end impedance

Consider voltage and currents at a point *P* located at a distance of d from the receiving end over a line, terminated over an impedance $\mathbf{Z}_l$, carrying a current $\mathbf{I}_l$ with a voltage $\mathbf{V}_l$ across, leading to $\mathbf{V}_r = \mathbf{V}_l$, $\mathbf{I}_r = \mathbf{I}_l$ and $\mathbf{Z}_r = \mathbf{Z}_l$ in Eq. (13.51). Recognizing the first term as incident wave and the second term as reflected wave, one can write that the RC at the point, *P* as,

$$\mathbf{\Gamma} = \frac{\mathbf{V}_{ref}}{\mathbf{V}_{inc}} = \frac{(\mathbf{Z}_r - \mathbf{Z}_o)e^{-\gamma d}}{(\mathbf{Z}_r + \mathbf{Z}_o)e^{\gamma d}} = \frac{(\mathbf{Z}_r - \mathbf{Z}_o)}{(\mathbf{Z}_r + \mathbf{Z}_o)}e^{-2\gamma d}$$

These voltage and currents can be expressed in terms of this RC as,

$$\mathbf{V} = \frac{\mathbf{I}_r}{2}(\mathbf{Z}_r + \mathbf{Z}_o)e^{\gamma d}(1 + \mathbf{\Gamma}) \qquad \& \qquad \mathbf{I} = \frac{\mathbf{I}_r}{2\mathbf{Z}_o}(\mathbf{Z}_r + \mathbf{Z}_o)e^{\gamma d}(1 - \mathbf{\Gamma})$$

Right over the load, $d = 0$ and $\mathbf{Z}_r = \mathbf{Z}_l$ and hence, the RC exactly over the load, $\mathbf{\Gamma}_l$, becomes

$$\mathbf{\Gamma}_l = \frac{(\mathbf{Z}_l - \mathbf{Z}_o)}{(\mathbf{Z}_l + \mathbf{Z}_o)}e^{-2\gamma \mathbf{0}} = \frac{(\mathbf{Z}_l - \mathbf{Z}_o)}{(\mathbf{Z}_l + \mathbf{Z}_o)} \qquad (14.9)$$

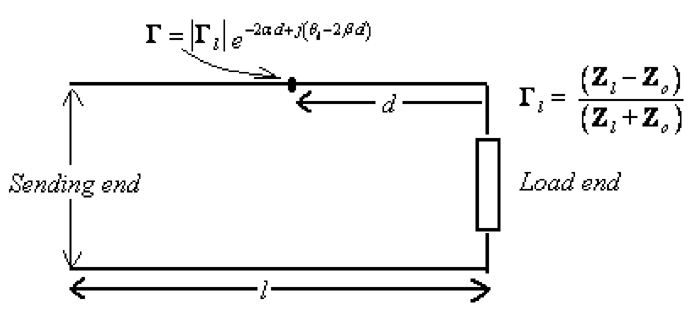


Figure 4 RC at load end and at an arbitrary point.

As $\mathbf{\Gamma}_l$ is a complex quantity, it can be written in terms of its magnitude, $|\mathbf{\Gamma}_l|$ and phase, θ_l as,

$$\mathbf{\Gamma}_l = |\mathbf{\Gamma}_l| e^{j\theta_l}$$

The $\mathbf{\Gamma}_l$ can also be expressed in terms of the normalized load impedance as,

$$\mathbf{\Gamma}_l = \frac{\mathbf{Z}_o(\mathbf{Z}_l/\mathbf{Z}_o - 1)}{\mathbf{Z}_o(\mathbf{Z}_l/\mathbf{Z}_o + 1)} = \frac{(\mathbf{z}_l - 1)}{(\mathbf{z}_l + 1)}$$

Here $\mathbf{z}_l$ is normalized load impedance. It is also possible to express the RC at a point P in terms of its value at load, as follows:

$$\mathbf{\Gamma} = \frac{(\mathbf{Z}_l - \mathbf{Z}_o)}{(\mathbf{Z}_l + \mathbf{Z}_o)} e^{-2\gamma d} = \mathbf{\Gamma}_l e^{-2\gamma d}$$

Substitution of $\mathbf{\Gamma}_l$ from Eq. (14.10) and $\boldsymbol{\gamma} = \alpha + j\beta$ in the above relation results in,

$$\mathbf{\Gamma} = |\mathbf{\Gamma}_l| e^{j\theta_l} e^{-2(\alpha + j\beta)d} = |\mathbf{\Gamma}_l|\, e^{-2\alpha d + j(\theta_l - 2\beta d)}$$

Hence, the RC $\mathbf{\Gamma}$ at a distance of d from the load end, is $\mathbf{\Gamma} = \mathbf{\Gamma}_l\, e^{-2\gamma d}$ with a magnitude, $\Gamma = |\mathbf{\Gamma}_l|\, e^{-2\alpha d}$ and with a phase of $(\theta_l - 2\beta d)$.

Note that one can obtain the same result using expressions for current, available in Eq.(13.51b). In case of loss-less line, the RC everywhere has the same magnitude including over the load. In case of lossy line, the reflected wave becomes smaller and the incident wave larger with increasing distance from the load causing $|\mathbf{\Gamma}|$ to decrease accordingly. Salient features and relations pertaining to RC are illustrated in Figure 14.3.

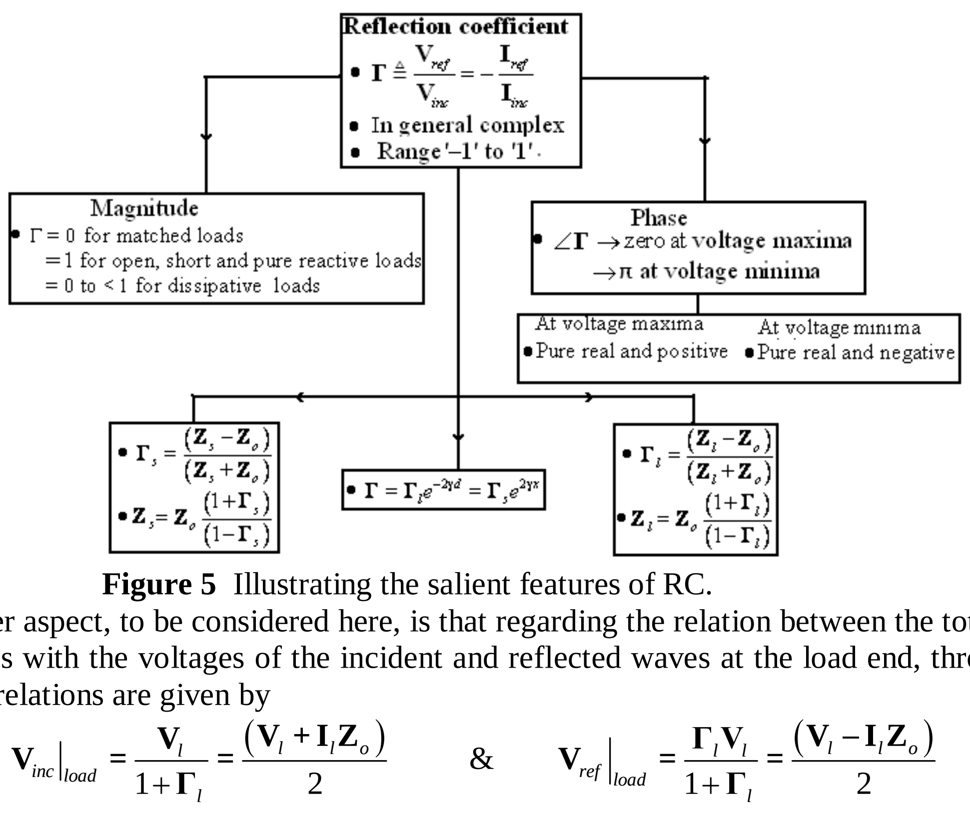


Figure 5 Illustrating the salient features of RC.

Another aspect, to be considered here, is that regarding the relation between the total voltage and currents with the voltages of the incident and reflected waves at the load end, through load RC. These relations are given by

$$\mathbf{V}_{inc}\big|_{load} = \frac{\mathbf{V}_l}{1 + \mathbf{\Gamma}_l} = \frac{(\mathbf{V}_l + \mathbf{I}_l \mathbf{Z}_o)}{2} \qquad \& \qquad \mathbf{V}_{ref}\big|_{load} = \frac{\mathbf{\Gamma}_l \mathbf{V}_l}{1 + \mathbf{\Gamma}_l} = \frac{(\mathbf{V}_l - \mathbf{I}_l \mathbf{Z}_o)}{2}$$

Derivation: Consider the total voltage across the load, which is sum of the voltages in the incident and reflected waves, and in terms of $\mathbf{\Gamma}_l$ it can be expressed as,

$$\mathbf{V}_l = \mathbf{V}_{inc}\big|_{load} + \mathbf{V}_{ref}\big|_{load} = \mathbf{V}_{inc}\big|_{load}\left(1+\mathbf{\Gamma}_l\right)$$

Rearranging the above relation results in,

$$\mathbf{V}_{inc}\big|_{load} = \frac{\mathbf{V}_l}{\left(1+\mathbf{\Gamma}_l\right)}$$

Substituting the expression for $\mathbf{\Gamma}_l$ available in Eq. (14.10) in the above expression, one can obtain,

$$\mathbf{V}_l = \mathbf{V}_{inc}\big|_{load}\left(1+\mathbf{\Gamma}_l\right) = \mathbf{V}_{inc}\big|_{load}\left[1+\frac{\left(\mathbf{Z}_l-\mathbf{Z}_o\right)}{\left(\mathbf{Z}_l+\mathbf{Z}_o\right)}\right] = \mathbf{V}_{inc}\big|_{load}\left[\frac{2\mathbf{Z}_l}{\left(\mathbf{Z}_l+\mathbf{Z}_o\right)}\right]$$

Thus, one can have incident voltage at load in terms of load voltage and currents as,

$$\mathbf{V}_{inc}\big|_{load} = \mathbf{V}_l\left(\frac{\mathbf{Z}_l+\mathbf{Z}_o}{2\mathbf{Z}_l}\right) = \left(\frac{\mathbf{V}_l\mathbf{Z}_l+\mathbf{V}_l\mathbf{Z}_o}{2\mathbf{Z}_l}\right) = \left(\frac{\mathbf{V}_l+\mathbf{I}_l\mathbf{Z}_o}{2}\right)$$

This completes the proof for first part of Eq.(14.13). By noting that reflected wave voltage is product of incident wave voltage with RC and following a procedure which is similar to the above, one can easily obtain the relation available in second part

Reflection Coefficient- Sending end impedance

Consider the expression for voltage and currents, at a point, *P* located at a distance of *x* from the sending end over the line. Recognizing the first term as incident wave and the second term as reflected wave, one can write the RC at the point, *P* is

$$\mathbf{\Gamma} = \frac{\mathbf{V}_{ref}}{\mathbf{V}_{inc}} = \frac{\left(\mathbf{Z}_s-\mathbf{Z}_o\right)e^{\gamma x}}{\left(\mathbf{Z}_s+\mathbf{Z}_o\right)e^{-\gamma x}} = \frac{\left(\mathbf{Z}_s-\mathbf{Z}_o\right)}{\left(\mathbf{Z}_s+\mathbf{Z}_o\right)}e^{2\gamma x}$$

The voltage and currents over the line can be expressed in terms of this RC as,

$$\mathbf{V} = \frac{\mathbf{I}_s}{2}\left(\mathbf{Z}_s+\mathbf{Z}_o\right)e^{-\gamma x}\left(1+\mathbf{\Gamma}\right) \qquad \& \qquad \mathbf{I} = \frac{\mathbf{I}_s}{2\mathbf{Z}_o}\left(\mathbf{Z}_s+\mathbf{Z}_o\right)e^{-\gamma x}\left(1-\mathbf{\Gamma}\right)$$

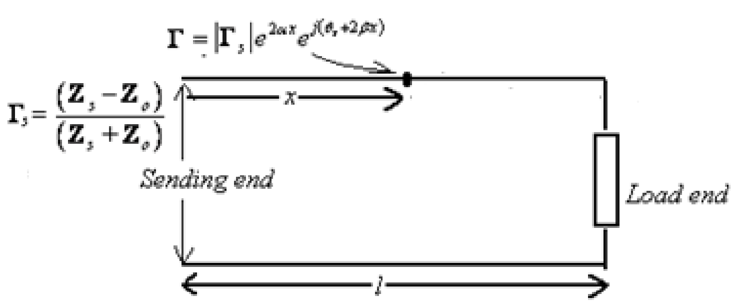


Figure 14.4 Reflection coefficient at sending end and at an arbitrary point.

At the sending end i.e. exactly over the source, *x* = 0 and the RC becomes,

$$\mathbf{\Gamma}_s = \mathbf{\Gamma}_{at\,x=0} = \frac{\left(\mathbf{Z}_s-\mathbf{Z}_o\right)}{\left(\mathbf{Z}_s+\mathbf{Z}_o\right)}$$

As $\mathbf{\Gamma}_s$ is a complex quantity and can have both magnitude, $|\mathbf{\Gamma}_s|$ as well as phase, θ_s. Hence,

$$\mathbf{\Gamma}_s = |\mathbf{\Gamma}_s| e^{j\theta_s}$$

The RC at the source end can also be expressed in terms of the normalized load impedance.

$$\mathbf{\Gamma}_s = \frac{\mathbf{Z}_o(\mathbf{Z}_s/\mathbf{Z}_o - 1)}{\mathbf{Z}_o(\mathbf{Z}_s/\mathbf{Z}_o + 1)} = \frac{(\mathbf{z}_s - 1)}{(\mathbf{z}_s + \mathbf{1})}$$

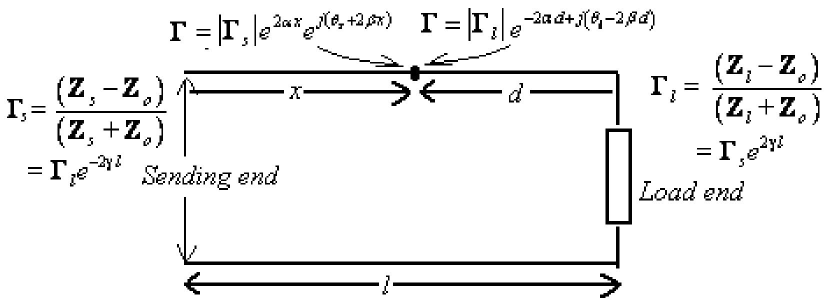


Figure 14.5 Inter-relation between RCs at different points over the line.

Then, RC at the point, P in terms of $\mathbf{\Gamma}_s$ becomes

$$\mathbf{\Gamma} = \frac{(\mathbf{Z}_s - \mathbf{Z}_o)}{(\mathbf{Z}_s + \mathbf{Z}_o)} e^{2\gamma x} = \mathbf{\Gamma}_s e^{2\gamma x}$$

Substitution of $\mathbf{\Gamma}_s$ from Eq.(14.16) and $\boldsymbol{\gamma} = \alpha + j\beta$ in Eq. (14.18a) results in,

$$\mathbf{\Gamma} = |\mathbf{\Gamma}_s| e^{j\theta_s} e^{2(\alpha + j\beta)x} = |\mathbf{\Gamma}_s| e^{2\alpha x} e^{j(\theta_s + 2\beta x)}$$

STANDING WAVE RATIO

The Standing Wave Ratio, or SWR, is one of the most important parameters used to describe, and also to quantify standing wave pattern over a loss-less line. It gives an indication of the amount of mismatch or reflected wave over the line. Note that this parameter is relevant only for loss-less lines.

SWR is defined as the ratio of maximum to minimum voltage in a standing wave pattern over a loss-less line. It can also be defined as the ratio of maximum to minimum current in standing wave pattern .However, it can be found that both are equal. SWR is denoted by 'ρ'(rho) and is always a dimensionless, pure real quantity, with value ranging from one to infinity. Mathematically, it can be defined as,

$$\rho \triangleq \frac{|\mathbf{V}_{\max}|}{|\mathbf{V}_{\min}|} = \frac{|\mathbf{I}_{\max}|}{|\mathbf{I}_{\min}|}$$

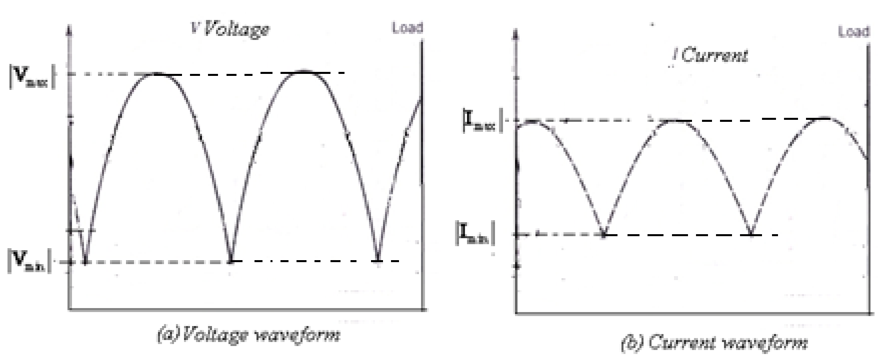


Figure 7 Definition of SWR. (a) Voltage waveform and (b) current waveform.

SWR can be considered as an indication/measure of amplitude ratio of reflected to incident waves. Thus, a value of unity for SWR denotes the absence of a reflected wave, while a very high SWR indicates that the reflected wave is as large as the incident wave. The SWR, can be shown, as infinite when the termination is open- or short-circuit or non-dissipative pure reactance.

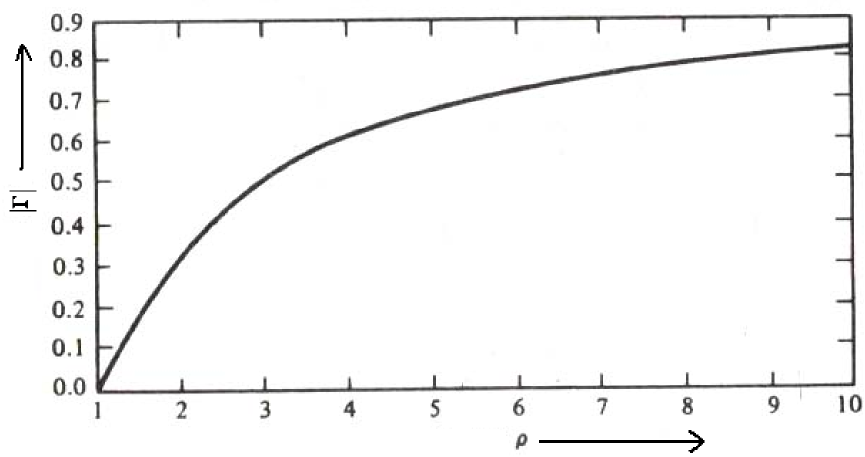


Figure 8 Relation between SWR and magnitude of RC.

SWR is related to RC and, in fact, it can be considered as a means of expressing the magnitude of the RC, 'Γ' when the line is loss-less. The exact analytical relation between the two can be specified either by expressing SWR in terms of Γ as,

$$\rho = \frac{1+|\mathbf{\Gamma}|}{1-|\mathbf{\Gamma}|} = \frac{1+\Gamma}{1-\Gamma}$$

Or by expressing Γ in terms of SWR as,

$$\Gamma = |\mathbf{\Gamma}| = \frac{\rho-1}{\rho+1}$$

Proof: SWR, by definition is ratio of $|\mathbf{V}_{\max}|$ to $|\mathbf{V}_{\min}|$. But voltage maximum occurs when incident and reflected waves interfere constructively, and hence, there the amplitude of the wave is sum of incident and reflected ones i.e. $|\mathbf{V}_{\max}|=|\mathbf{V}_{inc}|+|\mathbf{V}_{ref}|$. The voltage minimum occurs when incident and reflected waves interfere destructively, and hence, there the amplitude of the wave is difference of incident and reflected ones i.e. $|\mathbf{V}_{\min}|=|\mathbf{V}_{inc}|-|\mathbf{V}_{ref}|$. Introducing these two aspects in the defining relation,

$$\rho = \frac{|\mathbf{V}_{\max}|}{|\mathbf{V}_{\min}|} = \frac{|\mathbf{V}_{inc}|+|\mathbf{V}_{ref}|}{|\mathbf{V}_{inc}|-|\mathbf{V}_{ref}|} = \frac{1+|\mathbf{V}_{ref}|/|\mathbf{V}_{inc}|}{1-|\mathbf{V}_{ref}|/|\mathbf{V}_{inc}|} = \frac{1+|\mathbf{\Gamma}|}{1-|\mathbf{\Gamma}|} = \frac{1+\Gamma}{1-\Gamma}$$

The above relation can easily be manipulated to get that

$$(1-\Gamma)\rho = (1+\Gamma) \rightarrow (\rho-1) = \Gamma(\rho+1)$$

$$\Gamma = \frac{(\rho-1)}{(\rho+1)}$$

Note that, from the value of the RC, it is possible to find SWR. However, from SWR, it is possible to find only the magnitude of RC but not its phase.

SWR for different loads

The SWR assumes values depending upon the load. Its values for different types of loads are shown in Table 14.2. The necessary proofs are given hereunder.

Table 14.2 Values of SWR for various types of loads.

S.No.	Type of Load	SWR
1.	Matched termination	One (1)
2.	Open circuit or short circuit or pure reactance	Infinity(∞)
3.	Pure resistance	$\rho = \dfrac{\max(\mathbf{Z}_l, \mathbf{Z}_0)}{\min(\mathbf{Z}_l, \mathbf{Z}_0)}$
4.	Complex impedance	$\rho = \dfrac{1+\Gamma}{1-\Gamma}$, $\Gamma = \dfrac{\lvert \mathbf{Z}_l - \mathbf{Z}_0 \rvert}{\lvert \mathbf{Z}_l - \mathbf{Z}_0 \rvert}$

• When the termination is matched i.e. $\mathbf{Z}_l = \mathbf{Z}_o$, the SWR assumes a value equal to one.

Proof:

When the termination is matched(load matching is discussed completely in chapter 15), total power of the incident wave goes into the load, and as a result, the reflected wave becomes nil. As a consequence, only forward traveling wave exists over the line and as it is loss less, the amplitudes of the oscillations at all points over the line remain same. Hence, there exists no maxima points or minima points over the line, making $|\mathbf{V}_{max}|=|\mathbf{V}_{min}|$, resulting in SWR of unity.

• When the termination is pure reactance or open circuit or short circuit, the SWR is infinity.

Proof:

When the termination is one of the three categories i.e. pure reactance or open circuit or short circuit, then load can absorb no power from the incident wave. Thus, the entire incident wave gets reflected, the reflected wave is as strong as the incident wave. As the amplitudes of the incident and reflected waves are same, perfect cancellation takes place at the points of minima, making the minimum voltage $\mathbf{V}_{min}=0$, resulting in an SWR of infinity.

• When the termination is pure resistance i.e. $\mathbf{Z}_l = R_l$, then the SWR is either $\mathbf{Z}_o/R_l$ or $R_l/\mathbf{Z}_o$, which ever is more than one.

Proof:

The SWR is related to the magnitude of the RC through,

$$\rho = \frac{1+|\mathbf{\Gamma}|}{1-|\mathbf{\Gamma}|}$$

Even though the RC varies from point to point over the line, its magnitude remains same over the entire line, including the load point, provided the line is loss-less. Hence,

$$|\mathbf{\Gamma}| = |\mathbf{\Gamma}_l| = \left|\frac{\mathbf{Z}_l - \mathbf{Z}_o}{\mathbf{Z}_l + \mathbf{Z}_o}\right|$$

In the present case, where the line is loss less, the characteristic impedance $\mathbf{Z}_o$ is real and as line is terminated over a pure resistance, the load $\mathbf{Z}_l$ is also a real quantity. In addition, these two are positive quantities and under such circumstances,

$$|\mathbf{Z}_l - \mathbf{Z}_o| = \mathbf{Z}_l - \mathbf{Z}_o \text{ if } \mathbf{Z}_l > \mathbf{Z}_o$$

$$= \mathbf{Z}_o - \mathbf{Z}_l \text{ if } \mathbf{Z}_o > \mathbf{Z}_l$$

$$\left|\mathbf{Z}_l + \mathbf{Z}_o\right| = \mathbf{Z}_l + \mathbf{Z}_o$$

Substituting these values into the expression for SWR results in

$$\rho = \frac{1+|\boldsymbol{\Gamma}|}{1-|\boldsymbol{\Gamma}|} = \frac{1+|\boldsymbol{\Gamma}_l|}{1-|\boldsymbol{\Gamma}_l|} = \frac{|\mathbf{Z}_l + \mathbf{Z}_o| + |\mathbf{Z}_l - \mathbf{Z}_o|}{|\mathbf{Z}_l + \mathbf{Z}_o| - |\mathbf{Z}_l - \mathbf{Z}_o|}$$

Firstly, consider the case in which $\mathbf{Z}_l > \mathbf{Z}_o$::

$$\rho = \frac{|\mathbf{Z}_l + \mathbf{Z}_o| + |\mathbf{Z}_l - \mathbf{Z}_o|}{|\mathbf{Z}_l + \mathbf{Z}_o| - |\mathbf{Z}_l - \mathbf{Z}_o|} = \frac{\mathbf{Z}_l + \mathbf{Z}_o + \mathbf{Z}_l - \mathbf{Z}_o}{\mathbf{Z}_l + \mathbf{Z}_o - \mathbf{Z}_l + \mathbf{Z}_o} = \frac{\mathbf{Z}_l}{\mathbf{Z}_o}$$

Next, consider the situation where $\mathbf{Z}_o > \mathbf{Z}_l$::

$$\rho = \frac{|\mathbf{Z}_l + \mathbf{Z}_o| + |\mathbf{Z}_l - \mathbf{Z}_o|}{|\mathbf{Z}_l + \mathbf{Z}_o| - |\mathbf{Z}_l - \mathbf{Z}_o|} = \frac{\mathbf{Z}_l + \mathbf{Z}_o + \mathbf{Z}_o - \mathbf{Z}_l}{\mathbf{Z}_l + \mathbf{Z}_o - \mathbf{Z}_o + \mathbf{Z}_l} = \frac{\mathbf{Z}_o}{\mathbf{Z}_l}$$

Thus, the SWR is $\mathbf{Z}_o/R_l$ (for $\mathbf{Z}_o > \mathbf{Z}_l$) or $R_l/\mathbf{Z}_o$ (for $\mathbf{Z}_o < \mathbf{Z}_l$). In other words, SWR is one of these two ratios, whichever is more than one, and hence, mathematically, it can be expressed as,

$$\rho = \frac{\max(\mathbf{Z}_l, \mathbf{Z}_o)}{\min(\mathbf{Z}_l, \mathbf{Z}_o)}$$

• When the termination is complex impedance, then SWR can be found, first finding the magnitude of the RC over the line by using,

$$\Gamma = \frac{|\mathbf{Z}_l - \mathbf{Z}_o|}{|\mathbf{Z}_l - \mathbf{Z}_o|}$$

And then the SWR by using Eq. (14.41a). Or one can find it using the Smith chart also.

Significance of SWR

There is a lot of significance for SWR particularly at high frequencies like microwaves, where the lines are essentially loss-less. Its importance stems from the following facts:

- It is a quantity that is easily measureable, where as its competitor, RC, being complex, difficult to measure.
- It provides a means by which one can estimate the terminating impedance of loss less line.
- It can also a measure of the extent to which a reflected waves exist on the system.

SWR versus RC

Even though both these parameters are used to describe the mismatch and reflections over the line, there exist some fundamental and basic differences, listed below, between them.

1. RC can be defined both for lossy as well as loss-less lines, where as the standing wave ratio is meaningful only for loss-less lines.
2. RC varies from –1 to +1through 0, in general a complex quantity, whereas the standing wave ratio varies from 1 to ∞, always a real one.
3. RC varies from point to point over the line whereas the standing wave ratio is specified for the entire length of the line.
4. RC of voltages is negative of RC of currents whereas the standing wave ratio remains same whether it is voltage-ratio or current-ratio.
5. The chief advantage of SWR over the RC is its measurability. RC is difficult to measure where as the standing wave ratio is easily measurable quantity.

INPUT IMPEDANCE

The input impedance of a line is the impedance offered by it at the input terminals. As the source is connected at the input terminals, this quantity has some special significance while selecting the source. During computations, input impedance is quite an useful parameter to find the power flowing into the line when a generator is connected to it. To push maximum power over to the line, the source impedance and input impedance of line must have a complex conjugate relation.

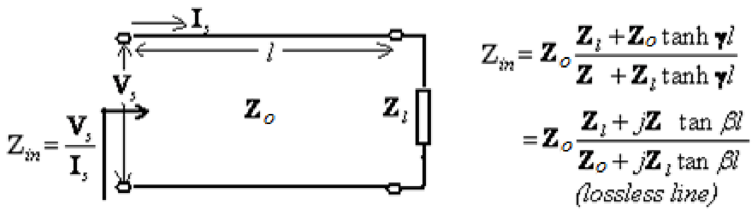


Figure 8 Input impedance of a line terminated over an impedance.

Formally, the input impedance of a line can be defined as the ratio of complex phasor voltage to complex phasor current at its input terminals, as shown in Figure 13.16. Mathematically,

$$\mathbf{Z}_{in} = \frac{\mathbf{V}_{in}}{\mathbf{I}_{in}} = \frac{\text{Input voltage}}{\text{Input current}}$$

It is complex quantity, value being dependent upon the configuration, length and termination of the line. For a line of length l terminated over an impedance, $\mathbf{Z}_l$, for a general line and then for a loss-less line, it is given by

$$\mathbf{Z}_{in} = \mathbf{Z}_o \frac{\mathbf{Z}_l + \mathbf{Z}_o \tanh \gamma l}{\mathbf{Z}_o + \mathbf{Z}_l \tanh \gamma l} \qquad \& \qquad \mathbf{Z}_{in} = \mathbf{Z}_o \frac{\mathbf{Z}_l + j\mathbf{Z}_o \tan \beta l}{\mathbf{Z}_o + j\mathbf{Z}_l \tan \beta l}$$

To derive these relations, consider a line of length l terminated over an impedance of $\mathbf{Z}_l$, The input impedance of this line by definition is

$$\mathbf{Z}_{in} = \mathbf{Z}\big|_{d=l} = \frac{\mathbf{V}_s}{\mathbf{I}_s} = \frac{\mathbf{V}}{\mathbf{I}}\bigg|_{d=l}$$

By substitution of the expressions for voltage and currents, from Eqs.(13.53), into the above relation, one can obtain that

$$\mathbf{Z}_{in} = \frac{\mathbf{V}_r \cosh \gamma d + \mathbf{I}_r \mathbf{Z}_o \sinh \gamma d}{\mathbf{I}_r \cosh \gamma d + (\mathbf{V}_r / \mathbf{Z}_o) \sinh \gamma d}\bigg|_{d=l}$$

$$= \frac{\mathbf{V}_r \cosh \gamma l + \mathbf{I}_r \mathbf{Z}_o \sinh \gamma l}{\mathbf{I}_r \cosh \gamma l + (\mathbf{V}_r / \mathbf{Z}_o) \sinh \gamma l} = \frac{(\mathbf{V}_r / \mathbf{I}_r) + \mathbf{Z}_o \tanh \gamma l}{1 + (1/\mathbf{Z}_o) \tanh \gamma l}$$

However, $\mathbf{V}_r / \mathbf{I}_r = \mathbf{Z}_l$, as the line is terminated over $\mathbf{Z}_l$. Thus,

$$\mathbf{Z}_{in} = \frac{\mathbf{Z}_l + \mathbf{Z} \tanh \gamma l}{1 + (1/\mathbf{Z}_o) \tanh \gamma l} = \mathbf{Z}_o \frac{\mathbf{Z}_l + \mathbf{Z}_o \tanh \gamma l}{\mathbf{Z}_o + \mathbf{Z}_l \tanh \gamma l}$$

In case of loss-less lines, $\gamma = j\beta$ and $\tanh \gamma l = j\tan\beta l$. In transmission line theory, the impedance of a line or load, divided by CI of line is called normalized impedance and process is named as normalization. Normalized impedances are indicated by small case letters. Input impedance can also be expressed in terms of normalized quantities:

$$\mathbf{z}_{in} = \frac{\mathbf{Z}_{in}}{\mathbf{Z}_o} = \frac{\mathbf{z}_l + \tanh \gamma l}{1 + \mathbf{z}_l \tanh \gamma l} \qquad \& \qquad \mathbf{z}_{in} = \frac{\mathbf{Z}_{in}}{\mathbf{Z}_o} = \frac{\mathbf{z}_l + j \tan \beta l}{1 + j\mathbf{z}_l \tan \beta l}$$

Similarly, expressions for input admittance, both for general as well lossy lines can be derived. It is also possible to express normalized input impedance and admittances in terms of normalized load impedance and admittances.

It can be very easily shown that input impedance is $\mathbf{Z}_o$ tanh γl in case of shorted line and $\mathbf{Z}_o$ coth γl in case of opened out line.

$$\mathbf{Z}_{in}\Big|_{\mathbf{Z}_l=0} = \mathbf{Z}_{sc} = \mathbf{Z}_o \left.\frac{\mathbf{Z}_l + \mathbf{Z}_o \tanh \gamma l}{\mathbf{Z}_o + \mathbf{Z}_l \tanh \gamma l}\right|_{\mathbf{Z}_l=0} = \mathbf{Z}_o \tanh \gamma l$$

With open circuit termination, its input impedance becomes,

$$\mathbf{Z}_{in}\Big|_{\mathbf{Z}_l\to\infty} = \mathbf{Z}_{oc} = \mathbf{Z}_o \left.\frac{\mathbf{Z}_l + \mathbf{Z}_o \tanh \gamma l}{\mathbf{Z}_o + \mathbf{Z}_l \tanh \gamma l}\right|_{\mathbf{Z}_l\to\infty} = \mathbf{Z}_o \coth \gamma l$$

In case of loss-less line, $\gamma = j\beta$ resulting in $\mathbf{Z}_{sc} = \mathbf{Z}_o\, j \tanh \beta l$ and $\mathbf{Z}_{oc} = -\mathbf{Z}_o\, j \coth \beta l$.

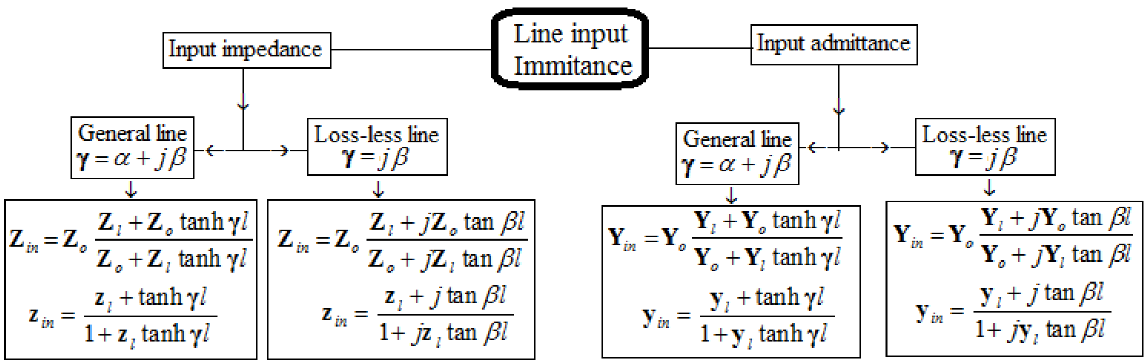
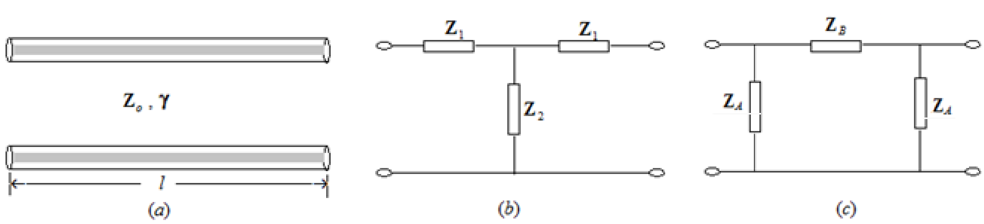


Figure 13.17 Input immitance of a transmission line.

T and π equivalent circuits:

Consider a T circuit with series elements equal to $\mathbf{Z}_1$ and shunt element equal to $\mathbf{Z}_2$. If this circuit is equivalent to a line of length, l then both must give same input impedance on short circuit as well on open circuit. Equating their input impedances on open circuit gives,



$$\mathbf{Z}_1 + \mathbf{Z}_2 = \mathbf{Z}_o \coth \gamma l \rightarrow \mathbf{Z}_2 = \mathbf{Z}_o \coth \gamma l - \mathbf{Z}_1$$

Equating their input impedances on short circuit gives,

$$\mathbf{Z}_1 + \mathbf{Z}_1 \,||\, \mathbf{Z}_2 = \mathbf{Z}_o \tanh \gamma l \rightarrow \mathbf{Z}_1(\mathbf{Z}_1 + \mathbf{Z}_2) + \mathbf{Z}_1\mathbf{Z}_2 = \mathbf{Z}_o(\mathbf{Z}_1 + \mathbf{Z}_2)\tanh \gamma l$$

Combining these two relations gives,

$$\mathbf{Z}_1\mathbf{Z}_o \coth \gamma l + \mathbf{Z}_1(\mathbf{Z}_o \coth \gamma l - \mathbf{Z}_1) = \mathbf{Z}_o\mathbf{Z}_o \coth \gamma l \tanh \gamma l = \mathbf{Z}_o^2$$

$$\mathbf{Z}_1^2 - 2\mathbf{Z}_1\mathbf{Z}_o \coth\gamma l + \mathbf{Z}_o^2 = 0 \rightarrow \mathbf{Z}_1 = \tfrac{1}{2}2\mathbf{Z}_o \coth\gamma l \pm \left[(2\mathbf{Z}_o \coth\gamma l)^2 - 4\mathbf{Z}_o^2\right]^{1/2}$$

$$\mathbf{Z}_1 = \mathbf{Z}_o \coth\gamma l \pm \left[(\mathbf{Z}_o \coth\gamma l)^2 - \mathbf{Z}_o^2\right]^{1/2} \rightarrow \mathbf{Z}_1 = \mathbf{Z}_o(\coth\gamma l \pm \operatorname{csch}\gamma l)$$

Solving this equation for $\mathbf{Z}_1$ results,

$$\mathbf{Z}_1 = \mathbf{Z}_o \tanh(\gamma l/2)$$

Now $\mathbf{Z}_2$ can be found from

$$\mathbf{Z}_2 = \mathbf{Z}_o \coth\gamma l - \mathbf{Z}_1 = \mathbf{Z}_o \coth\gamma l - \mathbf{Z}_o \tanh(\gamma l/2) = \mathbf{Z}_o/\sinh\gamma l$$

Consider a π circuit with shunt elements equal to $\mathbf{Z}_B$ and series element equal to $\mathbf{Z}_{A.}$. If this circuit is equivalent to a line of length, l then both must give same input impedance on short circuit as well on open circuit. Equating their input impedances on open circuit gives,

$$\mathbf{Z}_A \,||\, (\mathbf{Z}_A + \mathbf{Z}_B) = \mathbf{Z}_o \coth\gamma l$$

Equating their input impedances on short circuit gives,

$$\mathbf{Z}_A \,||\, \mathbf{Z}_B = \mathbf{Z}_o \tanh\gamma l$$

LINE SECTIONS

Small sections of lines find useful in the design of circuits for specific purposes like load measurement, impedance transformation, load matching etc., where they are not used for power or signal transmission. Eighth wave line, quarter wave line and half wave line belong to this category. A brief description and the theoretical background of these lines are given below.

13.10.1. Eighth-wave lines

The salient features of eighth wave transmission lines are:

- These are uniform and loss-less lines with real CI i.e. $\mathbf{Z}_o = R_o$ with length equal to $\lambda/8$.
- The magnitude of input impedance is equal to their CI, $|\mathbf{Z}_{in}|=R_o$ when termination is over a pure resistance, i.e. $\mathbf{Z}_l = R_l$

Proof: The input impedance, from Eq.(13.61), of a loss-less line of length $\lambda/8$ is

$$\mathbf{Z}_{in}\Big|_{l=\lambda/8} = \mathbf{Z}_o \left.\frac{\mathbf{Z}_l + j\mathbf{Z}_o \tan\beta l}{\mathbf{Z}_o + j\mathbf{Z}_l \tan\beta l}\right|_{l=\lambda/8} = \mathbf{Z}_o \frac{\mathbf{Z}_l + j\mathbf{Z}_o}{\mathbf{Z}_o + j\mathbf{Z}_l}$$

If the line is terminated in a pure resistance $\mathbf{Z}_l = R_l$, then,

$$\mathbf{Z}_{in}\Big|_{l=\lambda/8} = R_o \frac{R_l + jR_o}{R_o + jR_l}$$

The numerator and denominator have identical magnitudes, and hence, $|\mathbf{Z}_{in}|_{l=\lambda/8}=R_o$, a pure real quantity. Consequently, the magnitude of the input impedance becomes equal to the CI of line, $\mathbf{Z}_o$ (= R_o pure real as line is loss-less) when termination is over a pure resistance.

To conclude, an eighth-wave line can be used to transform any resistance into an impedance, hose magnitude is equal to CI i.e. $|\mathbf{Z}_{in}| = R_o$ of the line. Hence, it can be used to obtain a magnitude match between a load resistance of arbitrary value and a source of internal resistance equal to R_o .

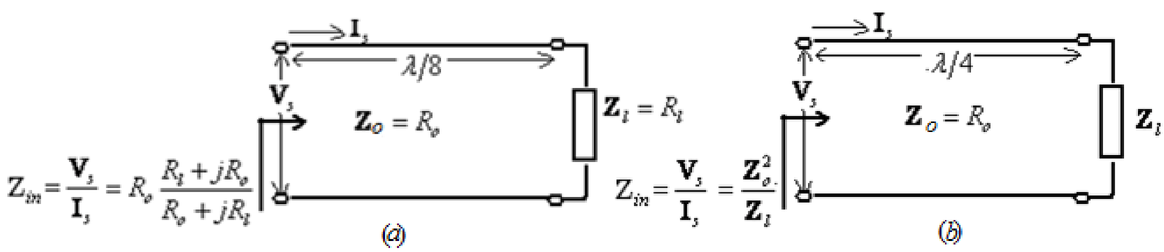


Figure 9 Input impedance of(a) eighth wave line and (b) quarter wave line.

Quarter-wave lines

The salient features of quarter wave transmission lines are:

- These are uniform loss-less lines with lengths given by $\lambda/4+n\lambda/2$, n = 0,1,2……
- The input impedance of these lines is inversely proportional to the terminating impedance

Proof: The input impedance, of a loss-less line of length $\lambda/4$ is

$$\mathbf{Z}_{in}\Big|_{l=\lambda/4} = \mathbf{Z}_o \frac{\mathbf{Z}_l + j\mathbf{Z}_o \tan\beta l}{\mathbf{Z}_o + j\mathbf{Z}_l \tan\beta l}\Bigg|_{l=\lambda/4}$$

As $\beta = 2\pi/\lambda$, $\beta l = (2\pi/\lambda)(\lambda/4) = \pi/2$

$$\mathbf{Z}_{in}\Big|_{l=\lambda/4} = \mathbf{Z}_o \frac{\mathbf{Z}_l + j\mathbf{Z}_o \tan(\pi/2)}{\mathbf{Z}_o + j\mathbf{Z}_l \tan(\pi/2)} = \mathbf{Z}_o \frac{\mathbf{Z}_l/\tan(\pi/2) + j\mathbf{Z}_o}{\mathbf{Z}_o/\tan(\pi/2) + j\mathbf{Z}_l} = \frac{\mathbf{Z}_o^2}{\mathbf{Z}_l}$$

The above relation can be put in an interesting form.

$$\frac{\mathbf{Z}_{in}\big|_{l=\lambda/4}}{\mathbf{Z}_o} = \frac{\mathbf{Z}_o}{\mathbf{Z}_l} \rightarrow \frac{1}{\mathbf{z}_l} = \mathbf{y}_l \rightarrow \mathbf{z}_{in}\big|_{l=\lambda/4} = \mathbf{y}_l$$

Thus, the normalized input impedance is equal to normalized load admittance for quarter wave section.

- From Eq. (13.81), it can be observed that the quarter wave section acts as an impedance transformer or impedance inverter. The input impedance is large when the terminal impedance is small and vice versa.

Provided the CI is resistive, a large pure resistance termination gets transformed into a small pure resistance and vice versa. Its input impedance is pure inductive if the termination is pure capacitive and vice versa. If the output impedance consists of a resistance in series with an inductive reactance, the input impedance becomes a resistance in parallel with a capacitive reactance and vice versa.

- An ideal quarter-wavelength line, $l=(2n-1)\lambda/4$, n=1,2,3.., is supposed to exhibit an input impedance of infinite on short circuit and zero on open circuit. However, infinite and zero impedances are not achievable in practice and what that appears is an input impedance of large value on short circuit and a small value on open circuit. Now, those impedances are estimated.

(1)Short circuited quarter-wavelength line, the input impedance is,

$$Z_{sc} = R_o/\alpha l$$

This is resistive, impedance maximum and highest value is possible when l is least achievable i.e. $l=\lambda/4$. The behavior of this line section can be found to similar to that of an antiresonant circuit near resonance.

(2)Open circuited quarter-wavelength line, $l=(2n-1)\lambda/4$ input impedance is

$$Z_{oc} = R_o\alpha l$$

This is resistive, impedance minimum and lowest value is possible when l is least feasible i.e. $l=\lambda/4$. The behavior of this line section can be found to similar to that of a series resonant circuit when frequency is varied near resonance.

Proof: Consider relation for input impedance for a shorted line of length l and expand it with $\gamma=\alpha+j\beta$:

$$\mathbf{Z}_{sc} = \mathbf{Z}_o \tanh\gamma l = \mathbf{Z}_o \frac{\sinh(\alpha + j\beta)l}{\cosh(\alpha + j\beta)l} = \mathbf{Z}_o \frac{\sinh\alpha l\cos\beta l + j\cosh\alpha l\sin\beta l}{\cosh\alpha l\cos\beta l + j\sinh\alpha l\sin\beta l}$$

For line of lengths equal to an odd multiple of a quarter wavelength, sin βl=±1and cos βl=0. As the attenuation is small when working at high frequencies, αl is also small, and in general, coshαl≈1, sinhαl≈ αl making the input impedance,

$$Z_{sc} \approx \mathbf{Z}_o \frac{\cosh \alpha l}{\sinh \alpha l} = \frac{R_o}{\alpha l}$$

When the line is on open circuit, $\mathbf{Z}_{oc} = \mathbf{Z}_o \coth \gamma l$ and manipulation on similar lines leads to ().

• It can also be used to step up the voltage. As long as it is loss-less, the ratio between output and input voltages is just square root of ratio of output to input impedances that are being matched.

Derivation:

Consider the voltage equation, for loss-less line,. For a quarter wave line, $d=\lambda/4$, and the input voltage $\mathbf{V}_s$ becomes,

$$\mathbf{V}_s = |\mathbf{V}|_{d=\lambda/4} = j(\mathbf{V}_r / \mathbf{Z}_r)\mathbf{Z}_o = j\mathbf{I}_l\,\mathbf{Z}_o$$

and hence, the ratio of input to load voltages becomes,

$$\frac{\mathbf{V}_s}{\mathbf{V}_l} = j\frac{\mathbf{I}_l}{\mathbf{V}_l}\mathbf{Z}_o = j\frac{1}{\mathbf{Z}_l}\sqrt{\mathbf{Z}_l\mathbf{Z}_s} = j\sqrt{\frac{\mathbf{Z}_s}{\mathbf{Z}_l}}$$

The voltage step up i.e. the ratio of output voltage to input voltage, then, becomes

$$\left|\frac{\mathbf{V}_l}{\mathbf{V}_s}\right| = \sqrt{\frac{\mathbf{Z}_l}{\mathbf{Z}_s}}$$

In the case of open circuited quarter wave line, the step up is infinite but it is for an ideal case of absolute loss-less line. To find the exact step up for practical case of a low-loss line, it requires considering the equation which takes the losses and the consequent attenuation into account. Consider the voltage equation. As the line is open circuited, $\mathbf{I}_l$ =0= $\mathbf{I}_r$, second term becomes zero. As the line is quarter wave in length, $l=\lambda/4$ and $\gamma l=(\alpha l+j\beta l)=(\alpha l+j\beta\lambda/4)=(\alpha l+j\pi/2)$. Incorporating these aspects into the voltage equation results in

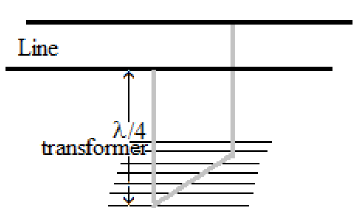
$$\mathbf{V} = \mathbf{V}_s = \mathbf{V}_r \cosh \gamma l = j\mathbf{V}_l \sinh \alpha l \approx j\mathbf{V}_l\, \alpha l$$

For a low- loss line, attenuation constant is given by, $\alpha=R/2\mathbf{Z}_o$ and for a one quarter wave line i.e. $l=\lambda/4$. Hence, voltage step-up is,

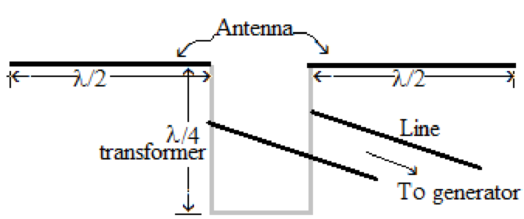
$$\left|\frac{\mathbf{V}_l}{\mathbf{V}_s}\right| \approx \frac{1}{\alpha l} = \frac{2\mathbf{Z}_o}{Rl}$$

Incidentally, for a three quarter-wave section, $l=3\lambda/4$, three times that of a single quarter wave section, and hence, the voltage step-up is one third of that for single quarter wave section.

• The frequency sensitivity is the main drawback of this line section.



(*a*)



(*b*)

Figure 10 Quarter-wave line in action. As (a) insulator and and for (b) driving (load matching)an antenna.

- ***Applications***:
  - It acts as *impedance transformer* or *inverter* as it can step-op or step-down the impedance.
  - It can be used as voltage step up transformer.
  - It is used for load matching purposes.
  - Another application of the sc quarter-wave line is as an insulator to support an open-wire line or the center conductor of a coaxial line. This application makes use of the fact that the input impedance of quarter-wave shorted line is very high.

13.10.3. Half-wave lines

The salient features of half wave transmission lines are:

- These are loss-less and uniform lines, with length given by, $n\lambda/2$, n=1,2……
- The input impedance of these lines is equal to the terminating impedance, as illustrated in Figure 13.21. This property is independent of CI, $\mathbf{Z}_o$ but frequency dependant.

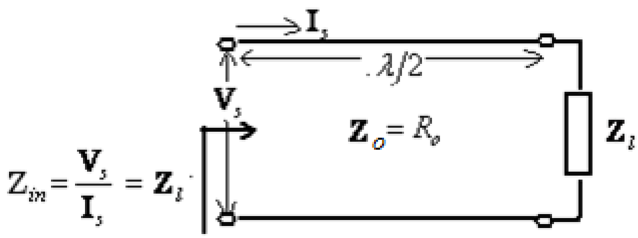


Figure 10 Input impedance of half-wave line.

Proof: The input impedance, from Eq.(13.61), of a loss-less line of length, $\lambda/2$ is

$$\mathbf{Z}_{in}\Big|_{l=\lambda/2} = \mathbf{Z}_o \frac{\mathbf{Z}_l + j\mathbf{Z}_o \tan\beta l}{\mathbf{Z}_o + j\mathbf{Z}_l \tan\beta l}\Bigg|_{l=\lambda/2} = \mathbf{Z}_o \frac{\mathbf{Z}_l + j\mathbf{Z}_o \tan\pi}{\mathbf{Z}_o + j\mathbf{Z}_l \tan\pi} = \mathbf{Z}_l$$

It proovess input impedance of $\lambda/2$ lines is equal to their terminating impedance.

- An ideal half-wavelength line, $l=n\lambda/2$, n=1,2,3.., is supposed to exhibit an input impedance of zero on short circuit and infinite on open circuit. However, zero and infinite impedances are not achievable in practice and what that appears is an input impedance of small value on short circuit and a large value on open circuit. Now, those impedances are estimated.

(1)Short circuited half-wavelength line, input impedance is

$$Z_{sc} = R_o \alpha l$$

This is resistive, impedance minimum and lowest value is possible when l is least feasible i.e. $l=\lambda/2$. The behavior of this line section can be found to similar to that of a series resonant circuit near resonance.

(2)Open circuited half-wavelength line, $l=n\lambda/2$ input impedance is

$$Z_{oc} = R_o / \alpha l$$

This is resistive, impedance maximum and highest value is possible when l is least achievable i.e. $l=\lambda/2$. The behavior of this line section can be found to similar to that of a parallel resonant circuit when frequency is varied near resonance.

Proof: Consider relation for input impedance for a shorted line of length l and expand it with $\gamma=\alpha+j\beta$, resulting in Eq. (13.86). For line of lengths equal to an even multiple of a half-wavelength, sin βl=0and cos βl=±1. As attenuation is small when working at high frequencies, αl is also small, and in general, coshαl≈1, sinhαl≈ αl making the input impedance,

$$Z_{sc} \approx \mathbf{Z}_o \frac{\sinh \alpha l}{\cosh \alpha l} = R_o \alpha l$$

It can be noticed that short circuited λ/4 line gives higher input impedance compared to open circuited λ/2 line and open circuited λ/4 line gives lower input impedance compared to short circuited λ/2 line. It implies that the line which is shortest gives better impedance properties and hence is desirable. The factor n appearing in various expressions also influences impedance, by lowering maxima and raising minima, as n increases.

Applications:

- It has its greatest utility in connecting a load to a source in situations where the load and source cannot be placed adjacent to each other.
- The short circuited λ/2 line can act as a band-stop filter, it can be used to measure velocity factor and dielectric constant of medium.
- Half wave line is also used to measure the impedance that is not accessible physically.

SPECIAL TYPE LOSS-LESS LINES

Loss-less transmission lines with short circuit or open circuit terminations are very often encountered, in applications like load measurement and load matching. Through proper choice of the length of a short or open circuited line, it is possible to obtain substitutes for capacitors and inductors with any desired reactance. Such a practice, indeed, is common in the design of microwave circuits and high-speed integrated circuits because making an actual capacitor or inductor is often more difficult than making a shorted or opened out transmission line.

It is also possible to simulate resonant circuits, with shorted or opened out transmission line. These properties are utilized in the design of band pass and band stop filters at microwave frequencies. It should be noted that unlike an ideal LC circuit, the shorted line has an infinite number of resonances.

Here, these special types of lines are considered and examined. Let us consider a uniform loss-less line of length 'l' lying along x-axis, with input point at x=0 and load point at x=l. First the line is short circuited and then it is open circuited.

Short circuited line

Note that as the line is shorted, load impedance is zero, $\mathbf{Z}_l$ =0. The voltage, current waveforms and input impedance of these lines are shown in Figure 14.15.

- RC: The voltage RC over the load is

$$\mathbf{\Gamma}_l\Big|_{\mathbf{Z}_l=0} = \frac{\mathbf{Z}_l - \mathbf{Z}_o}{\mathbf{Z}_l + \mathbf{Z}_o}\Bigg|_{\mathbf{Z}_l=0} = -1$$

As the line is loss less, the magnitude of the RC remains same at all points over the line.

$$\Gamma = |\mathbf{\Gamma}_l| = |-1| = 1$$

However, the phase of the RC changes from point to point.

- SWR: As the line is loss-less, it can have the standing wave ratio, SWR. For this line, it can be computed from the available value of Γ, as shown below.

$$\rho = \frac{1+\Gamma}{1-\Gamma} = \infty$$

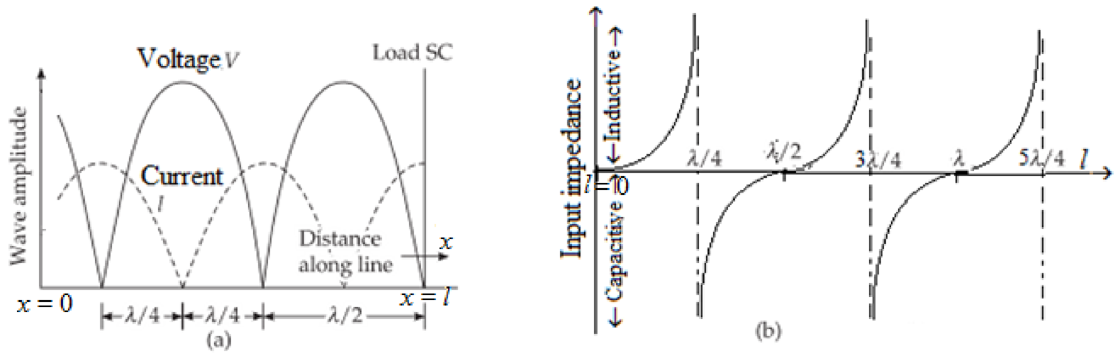


Figure 14.15. (a)Voltage and current waveforms and(b) input impedance of a line when shorted.

• Voltage pattern: The total voltage on the line is sum of voltages of incident and reflected waves, which can be expressed as

$$\mathbf{V} = \mathbf{V}^{+}e^{-j\beta x} + \mathbf{V}^{-}e^{j\beta x}$$

As line is loss-less and reflection is complete or perfect but with phase reversal(because of SC), $\mathbf{V}^{+} = -\mathbf{V}^{-}$. The load is located at $x=l$, and hence, the voltage pattern over a short circuited line becomes,

$$\mathbf{V}_{sc}(x) = -2j\mathbf{V}^{+}\sin\beta(x-l)$$

• Current pattern: The total current on the line can be expressed as

$$\mathbf{I} = \mathbf{Y}_o\left(\mathbf{V}^{+}e^{-j\beta x} - \mathbf{V}^{-}e^{j\beta x}\right)$$

With $\mathbf{V}^{+} = -\mathbf{V}^{-}$, the current pattern of shorted line becomes

$$\mathbf{I}_{sc}(x) = 2\mathbf{Y}_o\mathbf{V}^{+}\cos\beta(x-l)$$

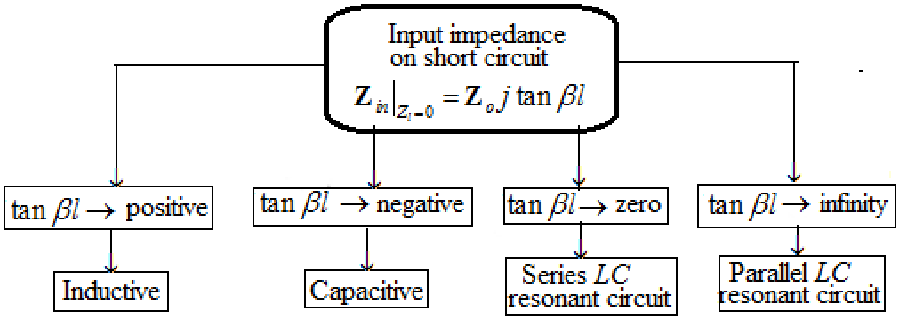
From the voltage pattern one can observe that the voltage is zero at the load i.e. at $x=l$ as it should be for a short circuit, and its amplitude varies as $\sin\beta l$. And from the current pattern, it can be observed that the current is maximum at the load and it varies as $\cos\beta l$.

In case of short circuit termination, it is a current anti-node and a voltage node that exists right over the load. In this case, as the total voltage is required to be zero over the load, the voltage must get reflected with 180^0 phase shift whereas the current need not under go any phase shift. It results in voltage node and current anti-node over the short circuit termination. The voltage and current waveforms on a short circuited loss-less line are shown in Figure 14.15(a).

• Input impedance: The input impedance of this loss-less shorted line can be computed as

$$\mathbf{Z}_{in}\big|_{Z_l=0} = \mathbf{Z}_o\, j\tan\beta l\,. \qquad (14.62)$$

From the above expression, input impedance of an open circuited line is purely reactive, and it can be positive, negative, zero and even tends to infinity, as is shown in Figure 14.15(b). Hence, it can behave like an inductor, capacitor and also as a resonant circuit.



Open circuited line

Note that as the line is open, load impedance tends to infinity, $\mathbf{Z}_l \rightarrow \infty$. The voltage, current waveforms and impedance of these lines are shown in Figure 14.16.

• RC: The voltage RC over the load is

$$\Gamma_l\Big|_{\mathbf{Z}_l \rightarrow \infty} = \frac{\mathbf{Z}_l - \mathbf{Z}_o}{\mathbf{Z}_l + \mathbf{Z}_o}\Bigg|_{\mathbf{Z}_l \rightarrow \infty} = 1$$

As the line is loss less, the magnitude of the RC remains same at all points over the line.

$$\Gamma = |\Gamma_l| = 1$$

However, the phase of the RC changes from point to point.

• SWR: As the line is loss-less, it can have the standing wave ratio, SWR. For this line, it can be computed from the available value of Γ, as shown below.

$$\rho = \frac{1+\Gamma}{1-\Gamma} = \infty$$

• Voltage pattern: The total voltage on the line is sum of incident and reflected waves, which can be expressed as,

$$\mathbf{V} = \mathbf{V}^+ e^{-j\beta x} + \mathbf{V}^- e^{j\beta x}$$

As line is loss-less and reflection is complete or perfect(because of OC), $\mathbf{V}^+ = \mathbf{V}^-$. The load is located at $x=l$, and hence, the voltage pattern over the open circuited line becomes,

$$\mathbf{V}_{oc}(x) = 2\mathbf{V}^+ \cos \beta (x - l)$$

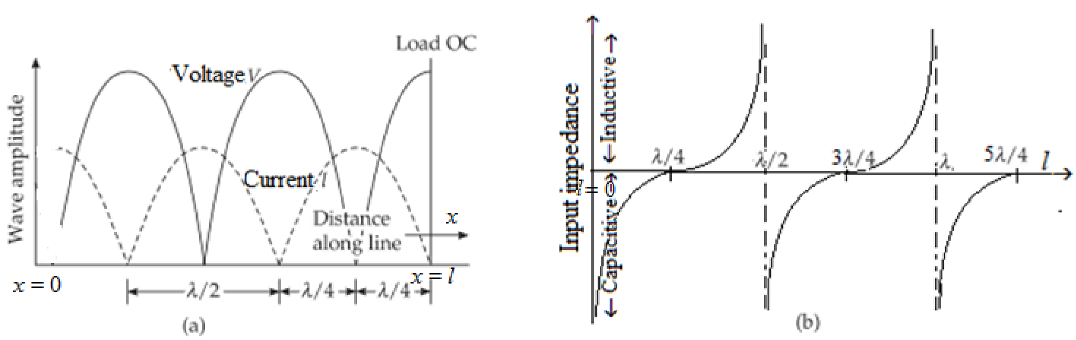


Figure 14.16. (a)Voltage and current waveforms and(b) input impedance of a line when open.

• Current pattern: The current on the line is

$$\mathbf{I} = \mathbf{Y}_o\left(\mathbf{V}^+ e^{-j\beta x} - \mathbf{V}^- e^{j\beta x}\right)$$

With $\mathbf{V}^+ = \mathbf{V}^-$, the current pattern over the line becomes,

$$\mathbf{I}_{oc}(x) = 2j\mathbf{Y}_o\mathbf{V}^+ \sin\beta(x-l)$$

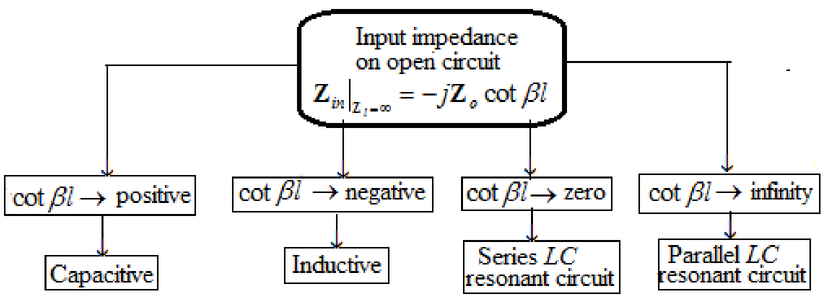
Note that from the current pattern, the current is zero at the load i.e. at $x = l$ as it should be for a open circuit, and its amplitude varies as sin βl. Similarly, from voltage pattern, it can be observed that the voltage is maximum over the load and it varies as cos βl.

When the termination is open circuit, the current gets reflected with 180^0 phase shift, since the total current has to be zero on an open circuit and the reflected current has to cancel the incident current which can happen only they are out of phase. However, the voltage gets reflected without any phase shift, as the direction of travel of the wave and phase of current being reversed, reflected voltage cannot have a phase shift. Thus, it is a current node and consequently, a voltage anti-node that exists right over the open circuit load.

Input impedance: The input impedance for a loss-less line is given by

$$\mathbf{Z}_{in}\big|_{\mathbf{Z}_l=\infty} = -j\mathbf{Z}_o \cot\beta l \,.$$

From the above expression, input impedance of an open circuited line is purely reactive, and it can be positive, negative, zero and even tends to infinity, as is shown in Figure 14.16(b). Hence, it can behave like an inductor, capacitor and also as a resonant circuit.



Some of of the applications of opened out lines in Antennas are,

- Opened out parallel wire λ/4 transmission line is used as wire radiator, called 'half wave dipole'.
- Opened out parallel wire transmission line of length less than λ/4 is used as wire parasitic radiator called 'director' in Yagi-Uda array. Thus, the director carries capacitive currents. In other words, an opened out line excited at a frequency less than resonant is capacitive.
- Opened out parallel wire transmission line of length more than λ/4 is used as wire parasitic radiator called 'reflector' in Yagi-Uda array. Hence, the reflectors carry inductive currents. In other words, an opened out line excited at a frequency more than resonant is inductive.

UNIT-II
TRANSMISSION LINES-II
Assignment-Cum-Tutorial Questions

SECTION-A

1. The distortion-less line condition is [ ]
   a) $R/L = G/C$ b) $R/L > G/C$
   c) $R/L < G/C$ d) None of these
2. The loading of line refers to the connection of [ ]
   a) Inductive coils b) Capacitive boxes
   b) Both (a) and (b) d) None of these
3. The reflection coefficient right over the source is [ ]
   a) $\frac{(\mathbf{Z}_s - \mathbf{Z}_o)}{(\mathbf{Z}_s + \mathbf{Z}_o)}$ (b) $\frac{(\mathbf{Z}_s + \mathbf{Z}_o)}{(\mathbf{Z}_s - \mathbf{Z}_o)}$
   (c) $\frac{(\mathbf{Z}_o - \mathbf{Z}_s)}{(\mathbf{Z}_o + \mathbf{Z}_s)}$ (d) None of these
4. The load voltage V_L of a lossless OC transmission line in terms of V^+ is [ ]
   a)$V_L = V^+$ b)$V_L = (½)V^+$ c) $V_L = 2\ V^+$ d) $V_L = (V^+)^2$
5. Z_0 in terms of Short circuit impedance Z_{Sc} and open circuit impedance Z_{Oc} [ ]
   a) $Z_0 = Z_{SC} * Z_{OC}$ b) $Z_0 = \sqrt{ZSC * ZOC}$ c) $Z_0 = (Z_{SC} * Z_{OC})^2$ d) $Z_0 = ½(Z_{SC} * Z_{OC})$
6. ------ Wave line acts as impedance transformer or inverter.
7. An open circuited line with length < λ/4 is equivalent to-------------.
8. An open circuited λ/4 line is equivalent to -----------------------circuit.
9. An open circuited line with length > λ/4 is equivalent to an------------.
10. The dependence of attenuation on frequency causes---------------------.
11. Distortion-less condition is ---.
12. Load matching refers to termination of line with -----------------------.
13. The input impedance of a transmission line is a function of its length. (yes/no)
14. Write the ranges of reflection coefficient and standing wave ratio.
15. Eighth-wave line transforms any resistance to impedance with a magnitude equal to ---------------------------------of the line.

SECTION-B
Descriptive questions

1. Explain about the λ/2, λ/4, λ/8 transmission lines. **[C02]**
2. Derive an expression for the input impedance of a loss-less line which it is terminated by (a) a load $\mathbf{Z}_l$ (b) open (c) short circuit and draw the suitable sketches. **[C02]**
3. Draw the line impedance curves of a lossless SC and OC transmission lines and analyze its inductive and capacitive properties. **[C02]**

4. Explain about the lossless lines and distortion less line. **[C01]**
5. What is loading? Discuss different types of loading methods mentioning their relative merits and demerits. **[C02]**
6. Describe the T and π sections of transmission line. **[C01]**
7. Differentiate SWR(S) from Reflection coefficient (K). **[C02]**
8. What is distortion-less condition? Derive the relation for distortion-less line condition on the primary constants. **[C02]**
9. What are properties and applications of eighth wave line, quarter wave line and half wave line? Given a list of their applications. **[C02]**

Problems

1. Determine the primary constants, R, L, G, and C for a distortion-less line working at 300MHz. Given that the line has characteristic impedance, Z_o =75Ω, attenuation constant, α=0.12Np/m, and wave velocity, v=1.4×10^8m/s. **[C02]**

2. A loss-less 75Ω line, 5λ/8 in length, is terminated on a load $\mathbf{Z}_l$. Find out its input impedance $\mathbf{Z}_{in}$ when (a) $\mathbf{Z}_l$= j45Ω (b) $\mathbf{Z}_l$= 25–j65Ω. **[C02]**

3. Determine the input impedance of a short circuited 50Ω coaxial line with β = 8.5rad/m when line length is (a)15cm(b)1.5m(c)3λ/4 and (d)λ/8. **[C02]**

4. A transmission line used to connect a transmitter to its antenna has a characteristic impedance Z_0 = 50 Ω. The antenna with impedance Z_L= (100+j75) Ω is connected as a load. Calculate load reflection coefficient. **[C01]**

5. A distortion less transmission line is characterized by R = 1.6Ω/m, L = 0.8µH/m, and C= 10 nF/m. Calculate shunt admittance G. **[C01]**

6. Z_{OC}= 900<-30^0,Z_{SC}=400<-10^o. Calculate the Z_0 and propagation constant of a 12 Km long line. **[C02]**

7. A distortion less line has Z_0=60 Ω, α=20 m Np/m, u=0.6c where c is the speed of the light in vacuum. Find R, L, G and C at 100MHz. **[C01]**

8. A lossless transmission line used in a TV receiver has a capacitance of 50 pF/m and an inductance of 200 nH/m. Find the characteristic impedance for sections of a line 10 meter long and 500 meter long. **[C02]**

SECTION-C

1. A transmission line with a characteristic impedance of 100Ω is used to match a 50 Ω section to a 200 Ω section. If the matching is to be done both at 429MHz and 1GHz, the length of the transmission line can be approximately(GATE2012) [ ]
   (A) 82.5cm (B) 1.05m **(C) 1.58m** (D) 1.75m
2. A transmission line of characteristic impedance 50Ω is terminated in a load impedance $\mathbf{Z}_l$. The VSWR of the line is measured as 5 and the first of the voltage maxima in the line is observed at a distance of λ/ 4from the load. The value of $\mathbf{Z}_l$ is (GATE2011)[ ]
   (A) 10 **(B) 250** (C) (19.23 + j46.15) (D) (19.23 – j46.15)

Transmission lines and Waveguides(UNIT III)

A. Questions testing the remembrance/understanding level of students

I. Objective/Multiple choice questions

1. In single stub matching, length of stub is --------- and location of stub is ---------
2. Equation representing circles is ------ and that representing arcs of the chart is ------
3. Double stub matching is not possible when the load falls in-------- region
4. Over the Smith chart, full circles represent------and arcs represent------.
5. Over the Smith chart, the upper half is------ and lower half is------.
6. Smith chart uses only------ and describe the line for ------.
7. The centre of constant SWR circle is always the------ of the chart.
8. V_{max}, I_{min} and ρ correspond to------ half the chart and I_{max}, V_{min} and $1/\rho$ correspond to------ half the chart

II. Descriptive questions

1. What is Smith chart.
2. Differentiate loss-less line from low-loss line.
3. Why CI is real and PC is imaginary at high frequencies.
4. Locate voltage max and min for pure resistive termination.

B. Questions testing the ability of students in applying the concepts

I. Multiple choice questions

1. The Smith chart can be characterized as
a) A Polar plot b) represents complex RC
c) Inscribed in a unity circle d) all
2. The complete circles and arcs in the Smith chart, respectively, represent

a) Normalized resistance/conductance, Normalized reactance/ susceptance
b)Normalized reactance/ susceptance, Normalized resistance/conductance
c) Normalized reactance, susceptance d) none of these
3. The circles and arcs over the Smith chart are
a) Orthogonal b) Opposite to each other
c) At 45^0 d) None of these
4. The upper half and lower half of the Smith chart, respectively, represent

a) Positive, nagative reactance/susceptances
b) Capacitive, inductive reactance/susceptances
c) Resistance, conductance d) None of these
5. The radius of the constant SWR circle is equal to

a) Voltage SWR b) Current SWR
c) Both (a) and (b) d) None of these
6. The centre of the constant SWR circle falls over

a) '1'of horizontal line b) centre of the chart
c) Both (a) and (b) d) None of these

7. In the left-half and right-half of the chart, resistance and reactance values, respectively, are
a) More than 1, less than 1 b) Less than 1, more than 1
c) 1,1 d) 0,0

8. The left most and rigtht most points of the chart, respectively, represent
a) (0,0), (∞,∞) b) (∞,∞),(0,0),
c) (0,0), (1,1) d) (1,1),(∞,∞)

9. The top most and bottom most points of the chart, respectively, represent
a) (1,1), (–1,–1) b) (–1,–1), (1,1)
c) (1,1), (0,0) d) None of these

10. Smith chart is always used with
a) Normalized impedances b) Normalized admittances
c) Both (a) and (b) d) None of these

11. The Smith chart is useful to analyze
a) Loss-less lines b) lossy-lines
c) Both (a) and (b) d) None of these

12. The horizontal line left and rigth of the centre, respectively, represent

a) $\mathbf{V}_{max}$, $\mathbf{I}_{max}$; $\mathbf{V}_{min}$, $\mathbf{I}_{min}$ b) $\mathbf{V}_{min}$, $\mathbf{I}_{max}$; $\mathbf{V}_{max}$, $\mathbf{I}_{min}$
c) $\mathbf{V}_{min}$, $\mathbf{I}_{min}$; $\mathbf{V}_{max}$, $\mathbf{I}_{max}$ d) None of these

13. The movement towards load and towards source over the line, respectively, correspond to,

a) Clock-wise, anticlock-wise rotation over the Smith chart
b) Anti-clockwise, clock-wise rotation over the Smith chart
c) Upwards, downwards d) None of these

14. The points over SWR circle, diametrically opposite to load impedance and load admittance points, respectively, are
a) Load admittance, load impedance b) Load impedance, load admittance
c) CI, CI d) None of these

15. Travel of length $\lambda_g/2$ over the line corresponds a rotation of
a) 180^0 over the chart b) 360^0 over the chart
c) 90^0 over the chart c) None of these

II. Problems

1. A loss-less $\mathbf{Z}_o$=100Ω line, terminated over an unknown impedance carries a wave with SWR=4. The first V_{min} is found at a distance of λ/8 from load. (a) Determine load impedance, $\mathbf{Z}_l$. When a matching QWT with CI, $\mathbf{Z}_{o1}$ is inserted, find out the (b) minimum distance between load and quarter wave line and (c)value of $\mathbf{Z}_{o1}$ in terms of $\mathbf{Z}_o$. Rework the problem when it is V_{max} at λ/8 instead of V_{min}.
***Answers*:** First case: (a)(47.05– *j* 88.23)Ω, (b)0.125λ, (c) $\mathbf{Z}_o$/2Ω Second case: (a)(47.05+ *j* 88.23)Ω, (b)0.125λ, (c)2$\mathbf{Z}_o$Ω

2. A loss-less air dielectric 120Ω line, working at 300MHz is terminated on a 36Ω resistive load. Find the length of a single shorted parallel connected stub and the location nearest to load for matching the line.
Answers: 39.4cm, 8cm,

3. A voltage source, V_g =120V with internal impedance, Z_g =50Ω is energizing a 50Ω line, 0.64λ in length and terminated on a load with impedance, Z_l =75Ω. Compute the time-averaged power delivered to line by source.
***Answers*:** 34.56W

4. A loss-less 140Ω air dielectric line i.e. ε_r=1 is matched to a load of 280Ω at 200MHz by means of a parallel shorted stub, Find the (a) length of stub and its position nearest to load. If the frequency is changed to 220MHz, without altering the circuit in anyway, find the (b)VSWR on the main line. Re-work the problem when the line has an insulation dielectric constant of 2.25.
Answers: For ε_r =1(a) 22.80cm,22.65cm (b)1.22,for ε_r =2.25(a)15.20cm,15.20cm(b)1.23

5. A loss-less 75Ω line is terminated on a load with impedance, (100+*j*150)Ω. Using Smith chart, determine the distance from load where the line impedance is (22.50+*j*47.25)Ω.
***Answers*:** 0.4λ.

6. A loss-less 50Ω line is connected to an unknown impedance giving a maximum voltage of $V_{\max}$ =0.90V and a minimum voltage of $V_{\min}$ =0.45V over the line. When the load is replaced by a short, the shift in minima is found as 0.15λ towards source. Using Smith chart, find the load impedance.
***Answers*:** $\mathbf{Z}_l$=(50+ *j*32.5)Ω

C. Questions testing the Analyzing/evaluating/creative abilities of student

1. Explain the need and method of loading technique. Discuss different types of loading methods mentioning their relative merits and demerits.
2. Define matched line. What are the advantages of transmission over matched line? Explain why a matched line does not carry reflected wave.
3. Describe the procedure of load matching with quarter wave transformer for different types of loads. What are the advantages and short comings involved in this method?
4. Describe the method of single stub matching. Derive the relation for the length and location of the stub.
5. Describe Smith chart and its salient features.

D. Previous GATE/IES questions

1. For pure reactance and pure resistance loads, load points over the Smith chart, respectively, stay at,
a) At the periphery, over the horizontal line b) Over the horizontal line, At the periphery
c) In the lower half, At the periphery d) In the upper half, over the horizontal line
2. For a match terminated loss-less line, the location of load point over the Smith chart is
a) At centre b) At periphery
c) In the upper half d) In the lower half

LOAD MATCHING

A line is said to be matched to the load when the load accepts all the power that has been placed over the line by generator without any reflections back. Under matched conditions, therefore, the line carrys only the forward traveling wave, with no reflected wave. It is shown soon that it can happen only when the load impedance is equal to the characteristic impedance of the line. Mathematically, matched termination implys,

$$\mathbf{Z}_l = \mathbf{Z}_o \qquad (15.6)$$

Proof: Consider an infinite line, with its load, naturally located at infinity. When a wave is impressed at its the source end, it starts traveling towards the load. However, as the load, the generation point of the reflected wave is at infinity, the wave can never reach it, and hence, there can never be a reflected wave over an infinite line.

Next, consider a finite length line terminated over its characteristic impedance. As the input impedance of the infinite line is equal to its characteristic impedance, the finite line under consideration can be viewed as a line terminated over an infinite line, making the combination another infinite line. As infinite line cannot have reflected waves, the finite length line terminated over its characteristic impedance also cannot have reflected waves, and hence, is a matched line.

When the line is designed for transfer of power to the load, then matched termination brings several advantages like maximum power transfer, maximum efficiency, lesser peak voltages, lower flashover likelihood, elimination of modulation distortion etc. Regarding power flow over a loss-less line, the following points, in the light of load matching, should be noticed:

- For reflection-less transfer of power into load from line, load impedance must be equal to characteristic impedance of line
- When loss-less line is terminated over short or open or pure reactance, power into the line system from generator as well into load from line is nil.
- For maximum power transfer from source into line, the source impedance must be conjugate of input impedance of line.
- The entire output of source can be placed over the line only when its internal impedance is equal to characteristic impedance.
- The reflected wave from the load gets nullified by the source when its internal impedance is equal to characteristic impedance.
- When the line is loss-less, the power delivered to load is equal to power placed over the line by source. In case of lossy line, power to load is less than the power placed over the line by source

When the line is terminated over impedance which is different from its characteristic impedance, reflections occur resulting in an inefficient transmission system and also all the benefits mentioned above are absent. Hence, mismatched operation of the line is unwanted and to eliminate or reduce the reflected wave over the line certain measures, called load matching techniques, are developed. They are,

1. Quarter-wave transformer technique
2. Single-stub matching technique, and
3. Double-stub matching technique.

In all these techniques, the reflected wave is eliminated from line only on the source side of matching device. Remaing part of line carrys reflected wave, and hence, standing wave.Another aspect of importance is, the distance between matching device and load is always less than one half-wavelength. However, incase of inaccesability or presence of any physical obstruction, the point of insertion may be shifted to an interger number of half wavelengths towards source side. Now, a detailed description of these techniques is given below.

Example 15.3: A 100Ω line of 1km long is terminated over a 200Ω load. It is fed by a generator of voltage, 10V and internal impedance, 50Ω. Find the load voltage and load power when the wave velocity, $v=2\times10^8$m/s and frequency, $f= 2\times10^5$rad/s.

Solution: Given that, l = 1000m, $\mathbf{Z}_r = \mathbf{Z}_l$ =200Ω, and $\mathbf{Z}_o$ =100Ω. The phase shift constant can be computed as $\beta=\omega/v=2\times10^5/2\times10^8 = 10^{-3}$ rad/m, giving βl=1, tanβl= tan1= 1.557. By substituting the available values in the expression, the input impedance can be obtained as,

$$\mathbf{Z}_{in} = 100\frac{200+j155.7}{100+j311.481} = 77.47\angle -0.598\Omega$$

This impedance is in series with the source resistance and the two together are across the voltage source. The current through this impedance gives sending end current and voltage across it gives sending end voltage of the line. They can be calculated as,

$$\mathbf{I}_s = \frac{\mathbf{V}_g}{\mathbf{Z}_g+\mathbf{Z}_{in}} = \frac{10\angle 0}{50\angle 0 + 77.47\angle -0.598} = 0.0819\angle 0.365\text{A}$$

$$\mathbf{V}_s = \mathbf{Z}_{in}\mathbf{I}_s = 77.47\angle -0.598\times 0.0819\angle 0.365 = 6.345\angle -0.233\,\text{V}$$

With the availability the sending end current and voltages, the receiving end current and voltages can be computed. In computing these quantities, the relation $\gamma x = j\beta x= j\times10^{-3}\times10^3 = j$, and also e^j=1∠1, e^{-j}=1∠–1 can be used.

$$\begin{aligned}\mathbf{V}_l &= \tfrac{1}{2}0.0819\angle 0.365\left[(77.47\angle -0.598+100\angle 0)1\angle -1+(77.47\angle -0.598-100\angle 0)1\angle 1\right]\\ &= 0.041\angle 0.365\left[(169.73\angle -1.26)+(56.54\angle -1.26)\right]\\ &= 9.27\angle -0.895\,\text{V}\end{aligned}$$

This is the value of the voltage across the load. The average power consumed in the load, then, becomes

$$p = \tfrac{1}{2}\left|\mathbf{V}_l\right|^2 / R_l = \tfrac{1}{2}\times 9.27^2 / 200 = 0.215\text{W}$$

Example 15.4: Given (a) $\mathbf{Z}_o$ = 100Ω, $\mathbf{Z}_l$=50Ω and (b) $\mathbf{Z}_o$ = 50Ω, $\mathbf{Z}_l$=100Ω, determine the time average power delivered to the load, when a loss-less line of length l =5λ/8 is connected to a source voltage, $\mathbf{V}_g$=100V, with an internal impedance, $\mathbf{Z}_g$=(30+j40)Ω.

***Solution*:**

For the given values, $\beta l=(2\pi/\lambda)\times(5\lambda/8)=1.25\pi$ rad and tan βl = tan 1.25π = 1. The input impedance can be obtained as,

$$\mathbf{Z}_{in} = 100\frac{50+j100}{100+j50} = (80+j60)\Omega$$

The current into the line is

$$\mathbf{I}_s = \frac{\mathbf{V}_g}{\mathbf{Z}_g+\mathbf{Z}_{in}} = \frac{100\angle 0}{30+j40+80+j60} = (0.497-j0.452) = 0.673\angle -0.74\text{A}$$

Now, the sending end voltage of the line is found.

$$\begin{aligned}\mathbf{V}_s = \mathbf{I}_s\mathbf{Z}_{in} &= (0.497-j0.452)(80+j60)\\ &= 66.88-j6.34 = 67.3\angle -0.09\,\text{V}\end{aligned}$$

As the line is loss-less, power into load is equal to the power into the line, which is product of sending end voltage and currents with power factor.

$$p_{ave} = \tfrac{1}{2}0.673\times 67.3\times \cos(0.65) = 17.99\text{W}$$

The angle, 0.65 (=0.74–0.09), in radians is the one in between sending end voltage and currents.
(b) In a similar manner, the input impedance and average power can be found as $(40 - j30)\Omega$ and 40W when Z_o = 50Ω, and Z_l=100Ω.

Example 15.5: Determine load impedance and time average power delivered under maximum power transfer conditions when a 100Ω loss-less line of length, l=0.2λ is driven by voltage source, $\mathbf{V}_g$=100V, with an internal impedance, $\mathbf{Z}_g$= (25+j50)Ω.

Solution:

Maximum power transfer into line happens when the source internal impedance is equal to complex conjugate of the input impedance of line. When maximum power enters into the line, as the line is loss-less, the power that enters into the load also becomes maximum. Hence, for maximum power transfer into line and then into load,

$$\mathbf{Z}_{in} = \mathbf{Z}_g^* = 25 - j50\Omega$$

From the given values, βl=(2π/λ)×0.2λ=0.4π rad and tan βl = tan 0.4π = 3.08. By substituting the available values in the expression for the input impedance, in Eq. (13.61), one can obtain,

$$25 - j50 = 100\frac{\mathbf{Z}_l + j100\times 3.08}{100 + j\mathbf{Z}_l\times 3.08} \rightarrow 0.25 - j0.50 = \frac{\mathbf{z}_l + j3.08}{1 + j\mathbf{z}_l 3.08}$$

$$(0.25 - j0.50)(1 + j\mathbf{z}_l 3.08) = (\mathbf{z}_l + j3.08)$$

$$\mathbf{z}_l(0.54 + j0.77) = (-0.25 + j3.58)$$

$$\mathbf{Z}_l = 100\frac{(-0.25 + j3.58)}{(0.54 + j0.77)} = (296.40 + j240.33)\Omega$$

Next, the power onto the line, which is equal to the power into the load from line can be computed. The current into the line is,

$$\mathbf{I}_s = \frac{\mathbf{V}_g}{\mathbf{Z}_g + \mathbf{Z}_{in}} = \frac{100\angle 0}{25 + j50 + 25 - j50} = 2\angle 0\ \text{A}$$

The sending end voltage of the line is,

$$\mathbf{V}_s = \mathbf{I}_s\mathbf{Z}_{in} = 2(25 - j50) = 111.8\angle -1.11$$

$$p_{ave} = \tfrac{1}{2}\times 2\times 111.8\times \cos(1.11) = 49.71\text{W}.$$

Note that the angle, 1.11 in radians, is the angle between the voltage and current.

Example 15.6: A loss-less $\mathbf{Z}_o$ = 300 Ω line is connecting a 100V generator to a pair of antennas, each with an input impedance, Z_{ant} = 73Ω, through two branch lines, with $\mathbf{Z}_o$ = 300 Ω. The lengths of main and branch lines are same and equal to 3λ/8. Find the average power delivered to each one of the antennas.

***Solution*:**

From the given data, βl=2π×3λ/8λ=3π/4 rad and tan 3π/4 = –1. The input impedance of each branch line is computed using the relation given in Eq. (13.61).

$$\mathbf{Z}'_{in} = \mathbf{Z}_o \frac{\mathbf{Z}_l + j\mathbf{Z}_o \tan \beta l}{\mathbf{Z}_o + j\mathbf{Z}_l \tan \beta l} = 150 \frac{73 - j150}{150 - j73} = (118 - j92.55)\ \Omega$$

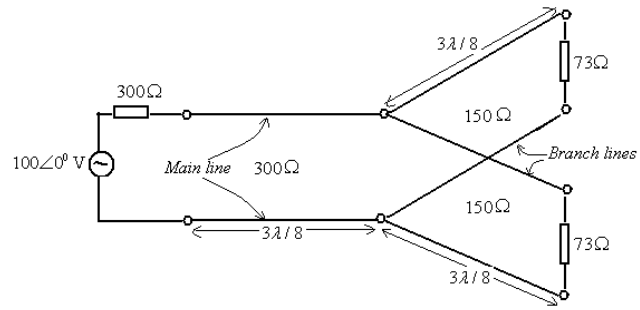


Figure 15.2 Generator feeding two antennas through two branch lines.

The load for main line is parallel combination of input impedances of branch lines, as shown in Figure 15.2, and it is given by,

$$\mathbf{Z}_L = \tfrac{1}{2}\mathbf{Z}'_{in} = 59 - j46.3\Omega$$

The input impedance of the main line then is

$$\mathbf{Z}_{in} = 300 \frac{(59 - j46.3) - j300}{300 - j(59 - j46.3)} = (156.53 - j373.10)\ \Omega$$

The input current to the main line is

$$I_{in} = \frac{100}{300 + (156.53 - j373.10)} = 0.169\angle 0.685\,\text{A}$$

The power into the main line gets divided equally between the branch lines, ultimately, to go into the antennas. Thus, the power delivered to each one of the antennas is half of the total power into the main line, which is given by

$$P_{av} = \tfrac{1}{2} I_{in}^2 R_{in} = \tfrac{1}{2}(0.169)^2 (156.53) = 2.235\ \text{W}$$

Power to each antenna, therefore, is 2.235/2=1.12W

15.2.1. Quarter wave transformer

Load matching can be enforced with a λ/4 length loss-less line, called quarter wave transformer (QWT). The reflection-less property of this device for continous waves is achieved by adjusting reflections at two ends to balance out at the designated frequency. As given in Eq.(13.66a), the input impedance of a quarter wave line is inversely proportional to the termination impedance and directly proportional to the square of the characteristic impedance of the transformer. Mathematically,

$$\mathbf{Z}_{in}\Big|_{l=\lambda/4} = \mathbf{Z}^{\,2}_{0,trans} / .\mathbf{Z}_l \qquad (15.7)$$

where $\mathbf{Z}_{o,trans}$ is the characteristic impedance of the transformer line and $\mathbf{Z}_l$ is its termination impedance.

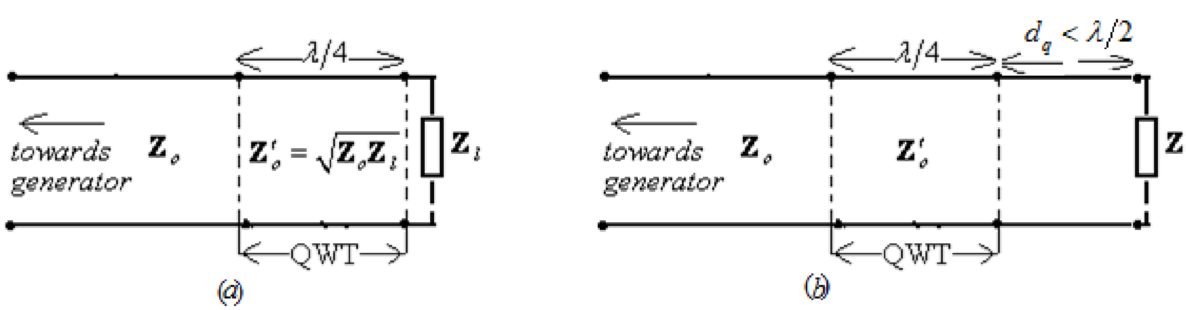


Figure 15.3 Load matching with Quarter-wave transformer.(a) Real resistive load and (b) complex impedance load.(a=13.1)

To match a loss-less line of characteristic impedance, $\mathbf{Z}_o$ to a load of $\mathbf{Z}_l \neq \mathbf{Z}_o$, a QWT is inserted between line and the load, as shown in Figure 15.3. The transformer is able to give the required matching, provided its characteristic impedance, $\mathbf{Z}_{o,trans}$ is related to $\mathbf{Z}_o$ and $\mathbf{Z}_l$ through

$$\mathbf{Z}_{o,trans} = \sqrt{\mathbf{Z}_o \mathbf{Z}_l} \tag{15.8}$$

Table 15.2 Details of QWT matching technique for complex loads.

S.No	Attribute	Load type	Minima	Maxima
1.	Location of QWT from load, d_q	Capacitive load: θ_l is negative	$-\frac{\lambda\theta_l}{4\pi}$	$\frac{\lambda}{4\pi}(-\theta_l+\pi)$
2.		Inductive load: θ_l is positive	$\frac{\lambda}{4\pi}(\theta_l+\pi)$	$\frac{\lambda\theta_l}{4\pi}$
3.	Characteristic impedance of QWT, $\mathbf{Z}_{o,q}$		$\mathbf{Z}_o\sqrt{\frac{(1-\Gamma_l)}{(1+\Gamma_l)}} = \mathbf{Z}_o\sqrt{\rho}$	$\mathbf{Z}_o\sqrt{\frac{(1+\Gamma_l)}{(1-\Gamma_l)}} = \frac{\mathbf{Z}_o}{\sqrt{\rho}}$

Proof: To avoid reflections, the termination impedance must be equal to the characteristic impedance of the line. The termination impedance of the line becomes the input impedance of QWT when the line is terminated over the transformer. The input impedance of the transformer terminated over $\mathbf{Z}_l$ can be varied and made equal to $\mathbf{Z}_o$ by appropriately selecting its characteristic impedance, $\mathbf{Z}_{o,trans}$. Then, that is under matching, it can be written that,

$$\mathbf{Z}_{in}\big|_{l=\lambda/4} = \mathbf{Z}^2_{o,trans}/\mathbf{Z}_l = \mathbf{Z}_o$$

It gives that

$$\mathbf{Z}^2_{o,trans} = \mathbf{Z}_o\mathbf{Z}_l \Rightarrow \mathbf{Z}_{o,trans} = \sqrt{\mathbf{Z}_o\mathbf{Z}_l}$$

It is same as Eq.(15.8). The procedure so far described works well only if the terminating impedance is purely resistive. Otherwise, $\mathbf{Z}_{o,trans}$, according to Eq.(15.8), becomes complex, which cannot be realized with a loss-less transformer line, whose characteristic impedance can assume only pure real values.

In case of complex load, this technique still can be used but with some minor modifications, which are mentioned below.

- First, convert the load impedance to admittance and then find the susceptance part of it.
- Tune out this susceptance part of load admittance by connecting a shorted stub across the load. Now, the effective load is pure real.
- Find the characteristic impedance of the transformer to be used, from the values of effective load impedance and the characteristic impedance of the line.
- Connect the quarter wave transformer in between the load and line.

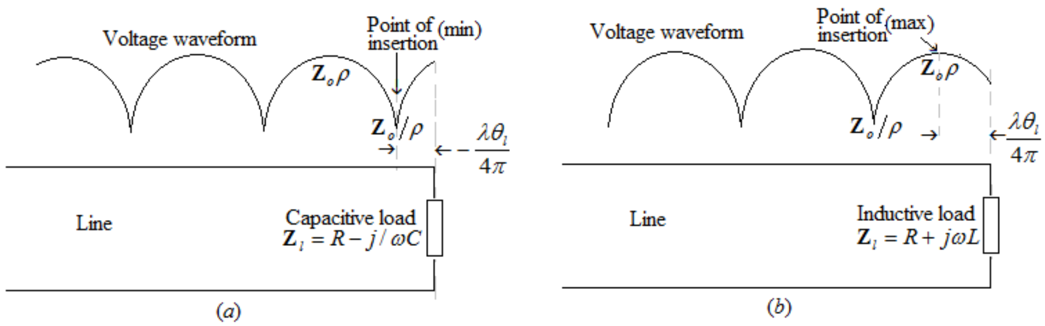


Figure 15.4 QWT insertion for load matching.(a) Capacitive load and (b) inductive load.

Another method, useful to match complex loads, involves insertion of a QWT at the location of a voltage maximum or a minimum point. It is already seen that the line impedance is pure real at these points, whose locations are available in Eqs. (14.35) and (14.36). It can be observed that at distance of $\lambda\theta_l/4\pi$, from the load, either a minimum or maximum occurs, depending upon the load type, as shown in Figure 15.4. It is preferred to insert the QWT at this point, because in such case, the least length of line remains under reflected wave.

The line impedance at minima is $\mathbf{Z}_o(1-\Gamma_l)/(1+\Gamma_l) = \mathbf{Z}_o/\rho$ and the characteristic impedance of the QWT to be inserted, therefore, should be $\mathbf{Z}_o\sqrt{(1-\Gamma_l)}/\sqrt{(1+\Gamma_l)} = \mathbf{Z}_o/\sqrt{\rho}$. And at maxima, which occurs at a distance of $\pm\lambda/4$ from the minima, the line impedance is $\mathbf{Z}_o(1+\Gamma_l)/(1-\Gamma_l) = \mathbf{Z}_o\rho$, and hence, the characteristic impedance of QWT should be $\mathbf{Z}_o\sqrt{(1+\Gamma_l)}/\sqrt{(1-\Gamma_l)} = \mathbf{Z}_o\sqrt{\rho}$. All the details pertaining to QWT matching technique for complex loads are given in Table 15.2

A short circuited λ/4 line is used for matching. The impedance at short circuit end is zero and at the other end it is infinity. In between the ends, impedance varies as sqaure of distance from shorted end. As shown in Figure 15.5(a), the load to be matched is connected across the output terminals, and the line to be matched is connected at a distance *d* from shorted end of λ/4 line where its impedance is equal to line impedance to be matched. When two different lines are to be matched to the same load, then the scheme shown in Figure 15.5(b) is suitable. For this technique, there is a limitation. At the input terminlas of a *practical* short circuited λ/4 line, input impedance is finite, not infinite. And hence, this system works well only when line to be matched has an impedance less than that which exists at the open circuit end of transformer.

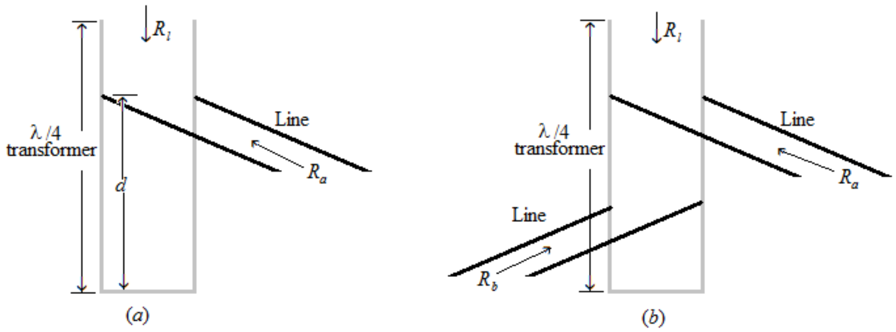


Figure 15.5 Load matching with tapped quarter-wave line.(a) single line R_a matching (b) two lines R_a , R_b matching to load, R_l

A serious drawback associated with this technique is due to QWT being a single frequency or narrow-band device, implying that this technique works satisfactorily only over a small band of frequencies. However, by transforming in smaller impedance steps, by using two or more quarter wave sections in series, each one accomplishing part of the total transformation, the bandwidth may be increased greatly. But it is the smooth and gradual transition gives maximum enhancement.

Another drawback is, it fails to function as a reflection suppressing device for a short pulse. Only for steady state condition or for a very long pulse, this device can provide a match. To avoid reflections for short pulses a gradually tapered line is needed.

Example 15.7: A quarter-wave transformer is used to match a 300Ω main line to a 200Ω secondary line terminated over its CI. When the system is working at 30MHz, assuming a velocity facator of 0.65, find physical length and CI of transformer. Also find VSWR over the main line when the transformer is not inserted.

Solution:

Assuming loss-less conditions, the physical length, *l* of λ/4 transformer can be computed as,

$$l = \frac{\lambda}{4} = \frac{0.65c}{4f} = \frac{0.65 \times 3 \times 10^8}{4 \times 30 \times 10^6} = 1.625\text{m}$$

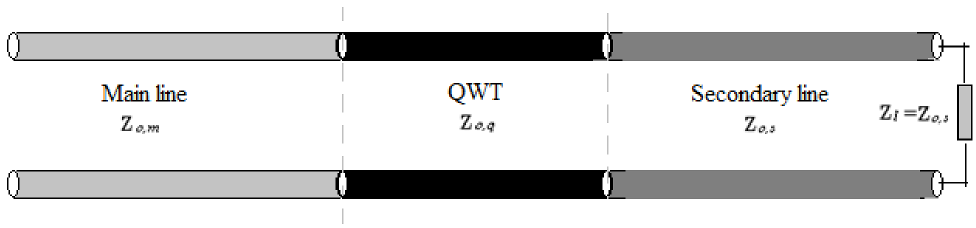


Figure 15.6 A secondary line feeding main line through a QWT.

The secondary line is terminated over its CI, and hence, its input impedance, which is also load of transformer, is same as its CI. As transformer is providing matching to main line, transformer input impedance must be CI of main line. Thus, transformer´s CI becomes,

$$\mathbf{Z}_{o,q} = \sqrt{\mathbf{Z}_{o,m}\mathbf{Z}_{in,s}} = \sqrt{300 \times 200} = 245\Omega$$

Without transformer, the load of main line is input impedance of secondary line, equal to 200Ω. Hence, VSWR becomes,

$$\rho = \frac{\text{Max}(R_o, R_l)}{\text{Min}(R_o, R_l)} = \frac{300}{200} = 1.5$$

Example 15.8: Determine CI and physical length of load matching QWT for the given data:

(a) line $\mathbf{Z}_o$= 100Ω, $\mathbf{Z}_l$= 150Ω,C=12pF, f=10MHz

(b) line $\mathbf{Z}_o$= 75Ω, $\mathbf{Z}_l$= 50Ω, ε_r=2.25, f=50MHz

(c) line $\mathbf{Z}_o$= 125Ω, $\mathbf{Z}_l$= 100Ω, velocity factor =0.8, f=100MHz

Solution:

(a) line $\mathbf{Z}_o$= 100Ω, $\mathbf{Z}_l$= 150Ω,C=12pF, f=10MHz

CI of QWT becomes,

$$\mathbf{Z}_{o,QWT} = \sqrt{\mathbf{Z}_o\mathbf{Z}_o} = \sqrt{100 \times 150} = 122.47\ \Omega$$

The physical length of QWT is,

$$l = \frac{\lambda}{4} = \frac{v}{4f} = \frac{1}{4fC\mathbf{Z}_o} = \frac{1}{4 \times 10 \times 10^6 \times 12 \times 10^{-12} \times 122.47} = 17.01\text{m}$$

(b) line $\mathbf{Z}_o$= 75Ω, $\mathbf{Z}_l$= 50Ω, ε_r=2.25, f=50MHz

CI of QWT becomes,

$$\mathbf{Z}_{o,QWT} = \sqrt{\mathbf{Z}_o\mathbf{Z}_o} = \sqrt{75 \times 50} = 61.24\ \Omega$$

The physical length of QWT is,

$$l=\frac{\lambda}{4}=\frac{v}{4f}=\frac{c}{4f\sqrt{\varepsilon_r}}=\frac{3\times10^8}{4\times10\times10^6\times\sqrt{2.25}}=5.00\text{m}$$

(c) line $\mathbf{Z}_o$= 125Ω, $\mathbf{Z}_l$= 100Ω, velocity factor =0.8, f=100MHz

CI of QWT becomes,

$$\mathbf{Z}_{o,QWT}=\sqrt{\mathbf{Z}_o\mathbf{Z}_o}=\sqrt{125\times100}=111.80\ \Omega$$

The physical length of QWT is,

$$l=\frac{\lambda}{4}=\frac{v}{4f}=\frac{0.8c}{4f}=\frac{0.8\times3\times10^8}{4\times100\times10^6}=0.60\text{m}$$

Example 15.9: Determine CI and physical length and physical distance of insertion from the load of a matching QWT for the given data:(C and ε_r pertains to QWT)

(a) line $\mathbf{Z}_o$= 100Ω, $\mathbf{Z}_l$= 50+j75Ω, C=12pF, f=10MHz.

(b) line $\mathbf{Z}_o$= 125Ω, $\mathbf{Z}_l$= 50–j75Ω, ε_r=2.25, f=50MHz.

Solution:

(a) line $\mathbf{Z}_o$= 100Ω, $\mathbf{Z}_l$= 50+j75Ω,C=12pF, f=10MHz.

RC over load and SWR over line can be found as,

$$\mathbf{\Gamma}=\frac{\mathbf{Z}_l-\mathbf{Z}_o}{\mathbf{Z}_l+\mathbf{Z}_o}=\frac{50+j75-100}{50+j75+100}=0.537\angle1.69$$

$$\rho=\frac{1+\Gamma}{1-\Gamma}=\frac{1+0.537}{1-0.537}=3.32$$

As load impedance is inductive, QWT insertion point is voltage maximum. Hence, characteristic impedance of QWT becomes,

$$\mathbf{Z}_{o,QWT}=\sqrt{\mathbf{Z}_o\mathbf{Z}_o\rho}=\mathbf{Z}_o\sqrt{\rho}=100\sqrt{3.32}=182.20\,\Omega$$

The physical length of QWT is,

$$l=\frac{\lambda}{4}=\frac{v}{4f}=\frac{1}{4fC\mathbf{Z}_o}=\frac{1}{4\times10\times10^6\times12\times10^{-12}\times182.20}=11.43\text{m}$$

Physical location of QWT is right over max whose distance from load, d_q is,

$$d_q=\frac{\lambda\theta_l}{4\pi}=\frac{v\theta_l}{4\pi f}=\frac{\theta_l}{4\pi fC\mathbf{Z}_o}=\frac{1.69}{4\pi\times10\times10^6\times12\times10^{-12}\times182.20}=6.15\text{m}$$

(b) line $\mathbf{Z}_o$= 125Ω, $\mathbf{Z}_l$= 50–j75Ω, ε_r=2.25, f=50MHz.

RC over load and SWR over line can be found as,

$$\mathbf{\Gamma}=\frac{\mathbf{Z}_l-\mathbf{Z}_o}{\mathbf{Z}_l+\mathbf{Z}_o}=\frac{50-j75-125}{50-j75+125}=0.557\angle-1.95$$

$$\rho=\frac{1+\Gamma}{1-\Gamma}=\frac{1+0.557}{1-0.557}=3.51$$

As load impedance is capacitive, QWT insertion point is voltage minimum. Hence, characteristic impedance of QWT becomes,

$$\mathbf{Z}_{o,QWT}=\sqrt{\mathbf{Z}_o\mathbf{Z}_o/\rho}=\mathbf{Z}_o/\sqrt{\rho}=125/\sqrt{3.51}=66.72\,\Omega$$

The physical length of QWT is,

$$l=\frac{\lambda}{4}=\frac{v}{4f}=\frac{c}{4f\sqrt{\varepsilon_r}}=\frac{3\times10^8}{4\times50\times10^6\sqrt{2.25}}=1.00\text{m}$$

Physical location of QWT is right over voltage min whose distance from load d_q is,

$$d_q=\frac{-\lambda\theta_l}{4\pi}=\frac{-c\theta_l}{4\pi f\sqrt{\varepsilon_r}}=\frac{3\times10^8\times1.95}{4\pi\times50\times10^6\sqrt{2.25}}=0.62\text{m}$$

Example 15.10: A loss-less $\mathbf{Z}_o$=125Ω line, terminated over an unknown impedance carries a wave with SWR=2.25. It is found that the first voltage maximum is at a distance of λ/8 from load. (a) Determine load impedance, $\mathbf{Z}_l$. For matching, a QWT with a CI of $\mathbf{Z}_{o1}$ is inserted. (b) Find out the minimum distance between load and quarter wave line and (c)value of $\mathbf{Z}_{o1}$ in terms of $\mathbf{Z}_o$. Rework the problem when it is first minimum at λ/8 instead of maximum.

Solution:

First V_{max} at λ/8:

$$\Gamma=\frac{\rho-1}{\rho+1}=\frac{2.25-1}{2.25+1}=0.385 \qquad \frac{\pi}{8}=\frac{\lambda\theta_l}{4\pi}\rightarrow\theta_l=1.57\text{rad}$$

$$\mathbf{Z}_l=\mathbf{Z}_o\frac{1+\mathbf{\Gamma}_l}{1-\mathbf{\Gamma}_l}=125\frac{1+0.385\angle1.57}{1-0.385\angle1.57}=(92.78+j83.87)\Omega$$

$$\mathbf{Z}_{o,QWT}=\sqrt{\mathbf{Z}_o\mathbf{Z}_o\rho}=\mathbf{Z}_o\sqrt{\rho}=125\sqrt{2.25}=\qquad\Omega$$

First V_{min} at λ/8:

$$\Gamma=\frac{\rho-1}{\rho+1}=\frac{2.25-1}{2.25+1}=0.385 \qquad \frac{\pi}{8}=\frac{-\lambda\theta_l}{4\pi}\rightarrow\theta_l=-1.57\text{rad}$$

$$\mathbf{Z}_l=\mathbf{Z}_o\frac{1+\mathbf{\Gamma}_l}{1-\mathbf{\Gamma}_l}=125\frac{1+0.385\angle-1.57}{1-0.385\angle-1.57}=(92.78-j83.87)\Omega$$

$$\mathbf{Z}_{o,QWT}=\sqrt{\mathbf{Z}_o\mathbf{Z}_o/\rho}=\mathbf{Z}_o/\sqrt{\rho}=125/\sqrt{2.25}=\qquad\Omega$$

Example 15.11: For a 75Ω loss-less line, find the location nearest to the load to insert QWT and the CI of QWT required to achieve matching for each of the following values of RC over the load, (a) $\mathbf{\Gamma}_l$ =1/9, (b) $\mathbf{\Gamma}_l$ = –j0.5 and (c) $\mathbf{\Gamma}_l$ =j/3.

Solution:

In the solution procedure, the first step is to ascertain whether it is minimum or maximum that occurs nearest to the load.

(a) In this case, over the load, the RC is pure real, and hence, its angle is zero i.e. θ_l = 0. It implies that load is pure real, voltage maximum or minimum occurs right over the load, depending upon $\mathbf{Z}_l > \mathbf{Z}_o$ or $\mathbf{Z}_l < \mathbf{Z}_o$. In both the cases, insertion point is right over the load. In the first case, maximum is right over the load, the CI of QWT can be computed as,

$$\mathbf{Z}_{o,q}=\mathbf{Z}_o\sqrt{\frac{(1+\Gamma_l)}{(1-\Gamma_l)}}=75\sqrt{\frac{[1+(1/9)]}{[1-(1/9)]}}=75\sqrt{\frac{10}{8}}=83.85\Omega$$

In the second case, minimum is right over the load, the CI of QWT can be computed as,

$$\mathbf{Z}_{o,q}=\mathbf{Z}_o\sqrt{\frac{(1-\Gamma_l)}{(1+\Gamma_l)}}=75\sqrt{\frac{[1-(1/9)]}{[1+(1/9)]}}=75\sqrt{\frac{8}{10}}=67.08\Omega$$

(b) In this case, over the load, the RC is negative and pure imaginary. And hence, the load is capacitive, nearest to load is minimum, where QWT can be inserted. Its CI is computed now:

$$\mathbf{Z}_{o,q} = \mathbf{Z}_o \sqrt{\frac{(1-\Gamma_l)}{(1+\Gamma_l)}} = 75\sqrt{\frac{[1-0.5]}{[1+0.5]}} = 75\sqrt{\frac{0.5}{1.5}} = 43.30\Omega$$

(c) In this case, over the load, the RC is positive and pure imaginary and hence, the load is inductive, nearest to load is maximum, where QWT can be inserted. Its CI is computed now:

$$\mathbf{Z}_{o,q} = \mathbf{Z}_o \sqrt{\frac{(1+\Gamma_l)}{(1-\Gamma_l)}} = 75\sqrt{\frac{[1+(1/3)]}{[1-(1/3)]}} = 75\sqrt{\frac{4}{2}} = 106.066\Omega$$

15.2.2. Single-stub matching

Stubs are small length, usually less than one half wavelength, loss-less lines with either an open ciruit or short circuit as termination. Short circuit is almost always preferred as open circuited stubs tend to radiate. These are widely used for load matching purpose. Their input admittance is pure susceptance, and depending upon the stub length, it can be inductive or capacitive. Stubs are used to nullify the line susceptance at the point of connection.When the line admittance is inductive susceptance, captive stub and in case of captive susceptance, inductive stub is used to nullify it.

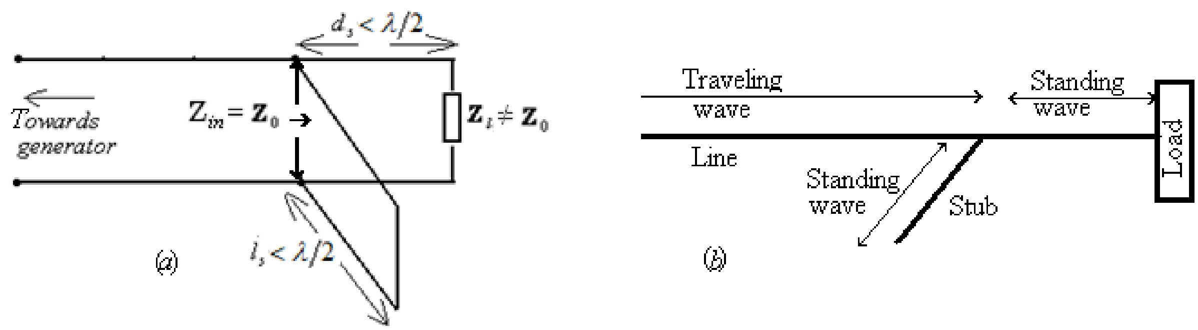
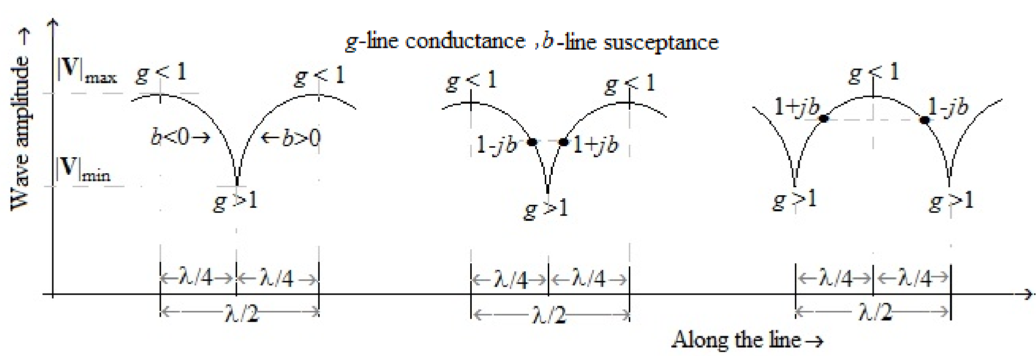


Figure 15.7 Load matching with a shorted single stub.(a) Stub connection and (b) type of waves over a stub matched line.(a=13.2modified)

Let us consider a piece of loss-less line, with characteristic impedance, $\mathbf{Z}_o$, terminated over a load, $\mathbf{Z}_l \neq \mathbf{Z}_o$. As the termination impedance is not equal to characteristic impedance, reflected wave, most of the times unwanted, comes into being over to the line. To eliminate the reflected wave, from a major portion of the line, several techniques are designed. One of such



method is single stub matching technique, involving connection of a short circuited stub of length, l_s to the line at a distance of d_s from the load side end, as shown in Figure 15.3.

Figure 15.8 Normalized line admittance in the vicinity of a voltage minimumm and maximum.

To understand the concept behind, consider a loss-less line and its normalized admittance in the neighborhood of a voltage minimumm and maximum, as shown in Figure 15.8. At $\mathbf{V}_{\min}$ and at $\mathbf{V}_{\max}$ normalized line admittance is pure conductive, g at the former it is more than one and at the later less than one. In between $\mathbf{V}_{\min}$ and $\mathbf{V}_{\max}$ at some point over the line, the conductivity assumes a value equal to one and hence, surrounding $\mathbf{V}_{\min}$ there exists two such points where g =1. As susceptance is positive on right side, negative on left side, the point on the right side can be specified as $1+jb$ and that on left side as $1-jb$. Also note that these two points, $1\pm jb$ are with in a distance of $\lambda/2$ from load.

A stub is connected at one of these two points, usually the one that is nearest to load, d_s selecting its length, l_s such that its susceptance is equal to that of the line at that point but opposite in sign, there by nullifying the line susceptance at that point. After stub connection, normalized line admittance is 1, denoting that line is matched.

Two possibilities arise, one pure resistive termination and the other complex impednace termination. Both are considered and analyzed.

Pure resistive load: The location of connection and length of stub are dependent upon the impedances of line and load. The *smallest* possible distance, d_s from the load side, where the stub can be attached is given by,

$$d_s = \frac{\lambda}{2\pi}\tan^{-1}\sqrt{\frac{\mathbf{Z}_l}{\mathbf{Z}_o}} \qquad (15.9)$$

The *shortest* possible stub length, l_s is given by,

$$l_s = \frac{\lambda}{2\pi}\left[n\pi \pm \tan^{-1}\left(\frac{\mathbf{Z}_o}{\mathbf{Z}_{os}}\frac{\sqrt{\mathbf{Z}_l\mathbf{Z}_o}}{\mathbf{Z}_l-\mathbf{Z}_o}\right)\right] \qquad (15.10)$$

Here, when $\mathbf{Z}_l > \mathbf{Z}_o$, n assumes zero value and sign is + and for $\mathbf{Z}_l < \mathbf{Z}_o$, n is 1 and sign is –. $\mathbf{Z}_{os}$ is CI of stub and when it is made from the same line, $\mathbf{Z}_{os} = \mathbf{Z}_o$. The above relations, in Eqs.(15.9) and (15.10), are valid only for purely resistive termination. The derivation of the above relations can be found in one of the solved Examples.

Complex impedance load: Most general situation is the one in which the line termination is a complex impedance. The relations, giving d_s and l_s, useful even for such a situations are available and given below.

The *smallest* possible distance, d_s from the load, where the shorted stub can be connected is given by,

$$d_s = \begin{cases} \dfrac{\lambda}{4\pi}\left(\theta_l \mp \dfrac{\pi}{2} \mp \tan^{-1}\dfrac{|b|}{2} - 2n\pi\right) \\ \dfrac{\lambda}{4\pi}\left(\theta_l \mp \pi \pm \cos^{-1}\Gamma_l - 2n\pi\right) \end{cases} \qquad (15.11)$$

Here, as b is the normalized susceptance of the stub, – b shall be that of the line at the point of stub connection. The upper sign is to be considered for positive values of b and the lower sign for its negative values. The angle, θ_l denotes the angle of the RC over the load. The integer, n can assume any value, positive or negative including zero, such that the distance of the stub is within a one half wavelength. The *shortest* length of stub, l_s can be found from,

$$l_s = \begin{cases} \dfrac{\lambda}{2\pi}\left(\pi - \tan^{-1}\dfrac{1}{b}\right) & \text{for } b > 0 \\ \dfrac{\lambda}{2\pi}\left(\tan^{-1}\dfrac{1}{|b|}\right) & \text{for } b < 0 \end{cases} \tag{15.12}$$

The value of b in terms of RC magnitude, Γ_l is

$$b = \pm\frac{2\Gamma_l}{\sqrt{1-\Gamma_l^2}} \tag{15.13}$$

Usually, Γ_l is given and from it b, d_s and l_s can be easily computed.

Now, the derivation of single stub technique relations pertaining to complex impedance termination is considered. Let us suppose stub of susceptance b is connected at point P located at a distance of d_s from load. After the attachment of stub, the normalized line admittance, **y** at connection point P must be equal to '1' for matching. However, before the connection of stub, the admittance of the line must be,

$$\mathbf{y} = 1 - jb$$

And the corresponding RC $\mathbf{\Gamma}_p$ i.e. at P is

$$\mathbf{\Gamma}_p = \frac{1-\mathbf{y}}{1+\mathbf{y}} = \frac{jb}{2-jb}$$

It is already seen that, for a loss-less line, the RC over the load $\mathbf{\Gamma}_l$ can be expressed in terms of RC over the line at the stub location, $\mathbf{\Gamma}_p$ as

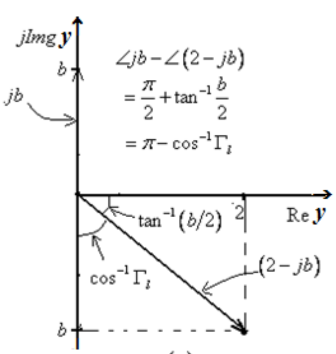
$$\mathbf{\Gamma}_l = \mathbf{\Gamma}_p e^{j\beta d_s} = \frac{jb}{2-jb}e^{j\beta d_s} = \frac{|b|}{\sqrt{4+b^2}}e^{j\left(\pm\pi/2 \pm \tan^{-1}|b|/2 + 2\beta d_s + 2n\pi\right)}$$

The phase of RC, $\mathbf{\Gamma}_p$ is equal to phase of its numerator ($\pm\pi/2$) subtracted by phase of the denominator ($\pm\tan^{-1}|b|/2 + 2n\pi$). In the above expression, the upper sign is to be considered for positive values of b and the lower sign for its negative values and n can assume positive or negative integer values. The RC magnitude, Γ_l over the load is,

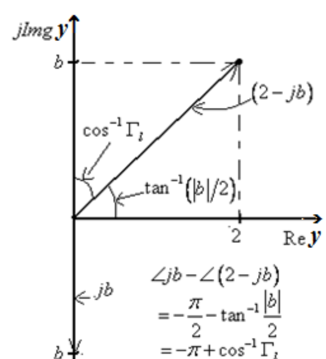
$$\Gamma_l = |\mathbf{\Gamma}_l| = \frac{|b|}{\sqrt{4+b^2}} \tag{15.14a}$$

Solving Eq. (15.14a) for b results in Eq. (15.13). The phase angle of RC is,

$$\theta_l = \left(\pm\frac{\pi}{2} \pm \tan^{-1}\frac{|b|}{2} + 2\beta d_s + 2n\pi\right) \tag{15.14b}$$



(a)



(b)

Figure 15.9 Pertaining to derivation of single stub matching. For (a) positive *b* and for (b) negative *b*.(13.3)

Solving Eq. (15.14b) for d_s results in upper part of Eq. (15.11). Using the information from Figure 15.9, the location of the stub can be expressed in terms of RC, as given in lower part of Eq. (15.11). Once d_s is available, length of the stub can be computed from the requirement of nullification of line susceptance by the stub for matching i.e. cot $\beta l_s = -b$. Solving this equation results in an expression for l_s as Eq. (15.13). The l_s can also be expressed in terms of Γ_l using Eq. (15.13) in Eq. (15.12). In the above relations, θ_l represents the angle of the RC over the load and Γ_l is the magnitude of the RC (magnitude of the RC over a loss-less line remains same independent of the location).

An important aspect of stub matching technique is the existence of reflected wave over various portions of line, in between source and the load. The reflected wave is absent only in between source and the point, where the stub is connected. However, it exists over the line in between point of stub connection and the load, incurring reflection losses, which depend upon length of line under reflected wave. And, the relation for stub location, available in Eq.(15.9) gives several points over the line, but to have a smallest possible line length under reflected wave, it is advisable to select a point which is nearest to the load.

The chief drawback of single stub technique is its narrow bandwidth. With change in frequency, the length and location of the stub has to be changed and it is the change of the location of the stub that is more troublesome. Another disadvantage is in the final adjustment the stub, which requires a very minute movement over the line. This is not possible for coaxial lines, resulting in ultimately an inaccurate matching.

To overcome these disadvantages, instead of one, two short circuited, position fixed stubs whose lengths are adjustable, independently, are used. The distance of the nearest stub from the load and the inter-stub distance is normally is either λ/4 or 3λ/8. The distance between the farthest stub and load, should be as small as possible so that a minimum possible length of line under reflected wave, incurring least amount of reflection losses.

Example 15.12: Derive the expressions for stub length and stub position when the load is pure resistance.

Solution:

Consider a loss-less line i.e. with pure real characteristic impedance $\mathbf{Z}_o = |\mathbf{Z}_o| = Z_o$ terminated over a pure resistancei.e.$\mathbf{Z}_l = |\mathbf{Z}_l| = Z_l$. Let us suppose stub is connected to line at point *P* which is at a distance of d_s from the load end. The normalized line impedance at this point is,

$$\mathbf{z}_p = \frac{\mathbf{Z}_p}{Z_o} = \frac{\mathbf{Z}_o}{Z_o}\frac{Z_l + jZ_o \tan\beta d_s}{Z_o + jZ_l \tan\beta d_s} = \frac{z_l + j\tan\beta d_s}{1 + jz_l \tan\beta d_s}$$

The corresponding normalized admittance is,

$$\mathbf{y}_p = \frac{1}{\mathbf{z}_p} = \frac{1 + jz_l \tan\beta d_s}{z_l + j\tan\beta d_s} = \frac{y_l + j\tan\beta d_s}{1 + jy_l \tan\beta d_s}$$

Let it be $g' + jb'$ and now, rationalizing $\mathbf{y}_p$ gives,

$$\mathbf{y}_p = \frac{y_l + j\tan\beta d_s}{1 + jy_l \tan\beta d_s}\frac{1 - jy_l \tan\beta d_s}{1 - jy_l \tan\beta d_s}$$

$$= \frac{y_l\left(1 + \tan^2\beta d_s\right) + j\left(1 - y_l^2\right)\tan\beta d_s}{\left(1 + y_l^2 \tan^2\beta d_s\right)}$$

By equating the real and imaginary parts results in

$$g' = \frac{y_l\left(1+\tan^2\beta d_s\right)}{\left(1+y_l^2\tan^2\beta d_s\right)} \qquad \& \qquad b' = \frac{\left(1-y_l^2\right)\tan\beta d_s}{\left(1+y_l^2\tan^2\beta d_s\right)}$$

If the normalized admittance $\mathbf{y}_p$ is made unity, indicating a match, then reflected wave over the line left of the point P disappears.The parameter $\mathbf{y}_p$ can be made unity i.e. $\mathbf{y}_p = g'=1$by proper selection ofd_s. The distance d_s of the point $P$from the load end which makes $g'=1$can be found from,

$$g' = \frac{y_l\left(1+\tan^2\beta d_s\right)}{\left(1+y_l^2\tan^2\beta d_s\right)} = 1 \rightarrow y_l\left(1+\tan^2\beta d_s\right) = \left(1+y_l^2\tan^2\beta d_s\right)$$

Solving this relation for d_s results,

$$\tan^2\beta d_s\left(y_l - y_l^2\right) = 1 - y_l$$

$$\tan^2\beta d_s = \frac{1-y_l}{y_l\left(1-y_l\right)} = \frac{1}{y_l} = \frac{Z_l}{Z_0} \rightarrow \beta d_s = \frac{\lambda}{2\pi}\tan^{-1}\sqrt{\frac{Z_l}{Z_o}}$$

Ultimately it results in an expression for d_s as,

$$d_s = \frac{\lambda}{2\pi}\tan^{-1}\sqrt{\frac{Z_l}{Z_o}}$$

The length of the short circuited stub which nullifys susceptance $b'$of the line can be found from the fact that, for nullification, its input admittance must be equal to $-b'$ .Assuming that the stub has characteristic impedance equal to that of the main line, the input admittance of a short circuited stub of length $l_s$is pure susceptance equal to,

$$\mathbf{Z}_{in}\Big|_{Z_l=0} = \mathbf{Z}_{sc} = Z_o\left.\frac{Z_l + jZ_o\tan\beta l_s}{Z_o + jZ_l\tan\beta l_s}\right|_{Z_l=0} = jZ_o\tan\beta l_s$$

The normalized admittance of stub, $\mathbf{y}_{sc}$ can be found as,

$$\mathbf{Y}_{sc} = \frac{1}{\mathbf{Z}_{sc}} = -jY_o\cot\beta l_s \rightarrow \mathbf{y}_{sc} = -j\cot\beta l_s$$

For nullification, the normalized admittance of stub, $\mathbf{y}_{sc} = -jb'$. Hence,

$$\cot\beta l_s = \frac{\left(1-y_l^2\right)\tan\beta d_s}{\left(1+y_l^2\tan^2\beta d_s\right)}$$

Substituting $\tan^2\beta d_s = z_l$ in the above relation

$$\cot\beta l_s = \frac{\left(1-y_l^2\right)\sqrt{z_l}}{\left(1+y_l^2 z_l\right)} = \frac{\left(1-y_l^2\right)\sqrt{z_l}}{\left(1+y_l\right)}$$

$$= \frac{\left(1-y_l\right)\left(1+y_l\right)\sqrt{z_l}}{\left(1+y_l\right)} = \left(1-y_l\right)\sqrt{z_l}$$

$$= \left(1-1/z_l\right)\sqrt{z_l} = \left(z_l-1\right)\sqrt{z_l}/z_l$$

$$= \left(z_l-1\right)/\sqrt{z_l} = \left(Z_l/Z_o-1\right)/\sqrt{Z_l/Z_o}$$

$$=\frac{(Z_l-Z_o)}{\sqrt{Z_lZ_o}}$$

Solving the above equation for l_s gives

$$\tan\beta l_s=\frac{\sqrt{Z_lZ_o}}{(Z_l-Z_o)}\rightarrow l_s=\frac{\lambda}{2\pi}\left(n\pi\pm\tan^{-1}\frac{\sqrt{Z_lZ_o}}{(Z_l-Z_o)}\right)$$

In the above expression, $n\pi$ is used as tan is periodic with π, and, as the requirement is shortest length, when $\mathbf{Z}_l > \mathbf{Z}_o$, n assumes zero value and sign is + and for $\mathbf{Z}_l < \mathbf{Z}_o$, n is 1 and sign is –.

When the stub used has different characteristic impedance, Z_{os} the input susceptance of the short circuited stub is equal to $-Y_{os}\cot\beta l_s$. If b' is the normalized susceptance then the susceptance of line at the stub connection point becomes Y_ob'. Thus, for matching,

$$Y_{os}\cot\beta l_s=Y_ob'=Y_o\frac{(1-y_l^2)\tan\beta d_s}{(1+y_l^2\tan^2\beta d_s)}$$

$$\cot\beta l_s=\frac{Y_o}{Y_{os}}\frac{(1-y_l^2)\tan\beta d_s}{(1+y_l^2\tan^2\beta d_s)}$$

Substituting $\tan^2\beta d_s= Z_l/Z_o = z_l$ in the above relation and then solving for l_s results in

$$l_s=\frac{\lambda}{2\pi}\left(n\pi\pm\tan^{-1}\frac{Z_o}{Z_{os}}\frac{\sqrt{Z_lZ_o}}{(Z_l-Z_o)}\right)$$

Example 15.13: A loss-less line, working at 100MHz with ε_r =2.1 and μ_r= 1, is to be matched to its load by means of a short circuited stub. Find the stub position closest to the load and its shortest length so that match is achieved when the characteristic and load impedances are (a) $\mathbf{Z}_o$ =50Ω, $\mathbf{Z}_l$ =100Ω and (b) $\mathbf{Z}_o$ =100Ω, $\mathbf{Z}_l$ =50Ω.

***Solution*:**

The wave velocity is,

$$v=\frac{c}{\sqrt{\varepsilon_r\mu_r}}=\frac{3\times10^8}{\sqrt{2.1}}=2.07\times10^8\,\text{m/sec}$$

The wave length, ratio of wave velocity to frequency, is

$$\lambda=2.07\times10^{10}/100\times10^6=207\text{cm}$$

(a) The distance of stub, d_s from the load end can be obtained as,

$$d_s=\frac{\lambda}{2\pi}\tan^{-1}\sqrt{\frac{\mathbf{Z}_l}{\mathbf{Z}_o}}=\frac{207}{2\pi}\tan^{-1}\sqrt{\frac{100}{50}}=32.9\times0.955=31.47\text{cm}$$

In this case, $\mathbf{Z}_o < \mathbf{Z}_l$, and hence, stub length, l_s can be found as,

$$l_s=\frac{\lambda}{2\pi}\tan^{-1}\frac{\sqrt{\mathbf{Z}_l\mathbf{Z}_o}}{\mathbf{Z}_l-\mathbf{Z}_o}=\frac{207}{2\pi}\tan^{-1}\frac{\sqrt{100\times50}}{100-50}=32.9\times0.955=31.47\text{cm}$$

(b)The distance of stub, d_s from the load end, can be obtained as,

$$d_s=\frac{\lambda}{2\pi}\tan^{-1}\sqrt{\frac{\mathbf{Z}_l}{\mathbf{Z}_o}}=\frac{207}{2\pi}\tan^{-1}\sqrt{\frac{50}{100}}=32.9\times0.6154=20.28\text{cm}$$

In this case, $\mathbf{Z}_o > \mathbf{Z}_l$, and hence, stub length, l_s can be found as,

$$l_s = \frac{\lambda}{2\pi}\left[\pi - \tan^{-1}\left(\frac{\sqrt{\mathbf{Z}_l\mathbf{Z}_o}}{\mathbf{Z}_l - \mathbf{Z}_o}\right)\right] = \frac{207}{2\pi}\left[\pi - \tan^{-1}\left(\frac{\sqrt{50\times100}}{50-100}\right)\right] = 103.5 - 31.47 = 72.04\text{cm}$$

Example 15.14: A loss-less 75Ω line with insulation dielectric constant, ε_r=2.25, is matched to a load of 100Ω at 200MHz by means of a parallel shorted stub. Determine (a) length of stub and its position nearest to load. If the frequency is changed to 250MHz, without altering the circuit in anyway, find the (b)VSWR on the main line

Re-work the problem, when CI is 100Ω and load impedance is 200Ω.

Solution:

Wavelengths corresponding to 200 MHz and 250 MHz are,

$$\lambda = \frac{3\times10^8}{200\times10^6\sqrt{2.25}} = 1\text{m} \qquad \& \qquad \lambda = \frac{3\times10^8}{250\times10^6\sqrt{2.25}} = 0.8\text{m}$$

Distance and lengths of the shorted parallel stub are,

$$d_s = \frac{\lambda}{2\pi}\tan^{-1}\sqrt{\frac{\mathbf{Z}_l}{\mathbf{Z}_o}} = \frac{1}{2\pi}\tan^{-1}\sqrt{\frac{100}{75}} = 13.64\text{cm}$$

$$l_s = \frac{\lambda}{2\pi}\tan^{-1}\frac{\sqrt{\mathbf{Z}_l\mathbf{Z}_o}}{\mathbf{Z}_l - \mathbf{Z}_o} = \frac{1}{2\pi}\tan^{-1}\frac{\sqrt{100\times75}}{100-75} = 20.50\text{cm}$$

After altering the frequency, the line impedance at stub conncted point is,

$$\beta d_s = \frac{2\pi}{\lambda}d_s = \frac{2\pi}{0.8}\times0.1364 = 1.071 \;\&\; \tan\beta d_s = 1.83$$

$$\mathbf{Z}_{in,l} = \mathbf{Z}_o\frac{\mathbf{Z}_l + \mathbf{Z}_o\, j\tan\beta d_s}{\mathbf{Z}_o + \mathbf{Z}_l\, j\tan\beta d_s} = 75\frac{100 + j75\times1.83}{75 + j100\times1.83} = 64.43\angle -0.24$$

The input impedance of stub is,

$$\beta l_s = \frac{2\pi}{\lambda}l_s = \frac{2\pi}{0.8}\times0.205 = 1.61 \;\&\; \tan\beta l_s = -25.45$$

$$\mathbf{Z}_{in,s} = j\mathbf{Z}_o\tan\beta l_s = j75\times(-25.45) = 1908.75\angle -1.57$$

The effective load impedance on the main line is parallel combination of line impedance and stub impedance. Hence,

$$\mathbf{Z}_l' = \mathbf{Z}_{in,l}\,||\,\mathbf{Z}_{in,s} = 64.43\angle -0.24\,||\,1908.75\angle -1.57 = 63.88\angle -0.27\,\Omega$$

The RC over the effective load impedance, and SWR over the line are,

$$\mathbf{\Gamma} = \frac{\mathbf{Z}_l' - \mathbf{Z}_o}{\mathbf{Z}_l' + \mathbf{Z}_o} = \frac{63.88\angle -0.27 - 75}{63.88\angle -0.27 + 75} = 0.158\angle -2.11$$

$$\rho = \frac{1+\Gamma}{1-\Gamma} = \frac{1+0.158}{1-0.158} = 1.37$$

When $\mathbf{Z}_o$=100Ω and $\mathbf{Z}_l$=200Ω, stub length and distances become, l_s=15.20cm, d_s=15.20cm with SWR =1.56

Example 15.15: A loss-less line of CI $\mathbf{Z}_o$ = 60Ω is to be matched to the load of 30Ω by means of a short circuited stub of same CI. Determine, using both sets of formulae, the stub position closest to the load and its length to obtain match.

***Solution*:**

The termination is pure resistance. The distance of stub, d_s from the load end, can be found as,

$$d_s = \frac{\lambda}{2\pi}\tan^{-1}\sqrt{\frac{\mathbf{Z}_l}{\mathbf{Z}_o}} = \frac{\lambda}{2\pi}\tan^{-1}\sqrt{\frac{30}{60}} = 0.098\lambda$$

The stub length, l_s can be found as

$$l_s = \frac{\lambda}{2\pi}\left[\pi - \tan^{-1}\left(\frac{\sqrt{\mathbf{Z}_l\mathbf{Z}_o}}{\mathbf{Z}_l - \mathbf{Z}_o}\right)\right] = \frac{\lambda}{2\pi}\left[\pi - \tan^{-1}\left(\frac{\sqrt{30\times 60}}{30-60}\right)\right] = 0.5\lambda - 0.152\lambda = 0.347\lambda$$

The above results can also be obtained using general relations. For the given values, RC over load can be found as,

$$\mathbf{\Gamma}_l = \frac{\mathbf{Z}_l - \mathbf{Z}_o}{\mathbf{Z}_l + \mathbf{Z}_o} = \frac{30-60}{30+60} = -\frac{1}{3} = 0.33\angle\pi$$

Susceptance of the stub can be obtained as,

$$b = \pm\frac{2\Gamma_l}{\sqrt{1-\Gamma_l^2}} = \pm\frac{2\times 0.333}{\sqrt{1-0.333^2}} = \pm 0.706$$

Distance of stub from load, can be computed as,

$$d_s = \frac{\lambda}{4\pi}\left(\pi \mp \frac{\pi}{2} \mp \tan^{-1}\frac{0.706}{2} - 2n\pi\right)$$
$$= \left(0.25 \mp 0.125 \mp 0.027 - 0.5n\right)\lambda$$
$$= \begin{cases}\left(0.098 - 0.5n\right)\lambda & \text{for } b > 0\\ \left(0.402 - 0.5n\right)\lambda & \text{for } b < 0\end{cases}$$

In the above expression, a value for n should be selected such that the distance is within one half wavelength. In the present case, the appropriate value is zero. Hence,

$$d_s = \begin{cases}0.098\lambda & \text{for } b > 0\\ 0.402\lambda & \text{for } b < 0\end{cases}$$

The smaller one of the above two is 0.098λ which happens when $b > 0$. The length of the stub, for such case can be found as,

$$l_s = -\frac{\lambda}{2\pi}\left(\tan^{-1}\frac{1}{0.706}\right) + \frac{\lambda}{2} = \frac{\lambda}{2\pi}\left(-0.956\right) + \frac{\lambda}{2} = 0.347\lambda$$

Notice, both sets of formulae give the same result.

Example 15.16: A loss-less 60Ω line is to be matched to a complex impedance load by means of a short circuited stub. Given that stubs are made from the main line and complex impedance load (a) $\mathbf{Z}_l$= (12– j24)Ω and (b) $\mathbf{Z}_l$= (12+j24)Ω. Determine the stub position closest to the load and its length so that match is obtained.

***Solution*:**

Given that stubs are made from the main line, and hence, their CIs is same as that of main line.Here the load is complex and the the required quantities can be found as follows:

(a) $\mathbf{Z}_l$ = (12– j24)Ω

For the given values, RC over load,

$$\mathbf{\Gamma}_l = \frac{\mathbf{Z}_l - \mathbf{Z}_o}{\mathbf{Z}_l + \mathbf{Z}_o} = \frac{(12 - j24) - 60}{(12 - j24) + 60} = 0.707\angle -2.356$$

Magnitude of the coefficient, Γ_l and its phase, θ_l are 0.707 and −2.356 rad. Susceptance of the stub can be obtained as,

$$b = \pm \frac{2\Gamma_l}{\sqrt{1-\Gamma_l^2}} = \pm \frac{2\times 0.707}{\sqrt{1-0.707^2}} = \pm 2.00$$

Distance of stub from load can be found as,

$$d_s = \frac{\lambda}{4\pi}\left(-2.356 \mp \frac{\pi}{2} \mp \tan^{-1}\frac{2.00}{2} - 2n\pi\right)$$
$$= \lambda\left(-0.187 \mp 0.125 \mp 0.0625 - 0.5n\right)$$
$$= \begin{cases} \left(-0.3745 - 0.5n\right)\lambda & \text{for } b > 0 \\ \left(0.0005 - 0.5n\right)\lambda & \text{for } b < 0 \end{cases}$$

In the above expressions, appropriate value for n is −1 for b>0 and 1 for b<0. Thus, distance of stub from load is,

$$d_s = \begin{cases} 0.1255\lambda & \text{for } b > 0 \\ 0.4995\lambda & \text{for } b < 0 \end{cases}$$

The length of the stub, for each case can be found as

$$l_s = \begin{cases} \frac{\lambda}{2\pi}\left(\pi - \tan^{-1}\frac{1}{2}\right) = \frac{\lambda}{2\pi}\left(\pi - 0.464\right) = 0.426\lambda & \text{for } b > 0 \\ \frac{\lambda}{2\pi}\left(\tan^{-1}\frac{1}{2}\right) = \frac{\lambda}{2\pi}\left(0.464\right) = 0.0738\lambda & \text{for } b < 0 \end{cases}$$

Closest postion and corresponding length of stub are, 0.1255λ and 0.426λ.

(b) $\mathbf{Z}_l = (12+j24)\Omega$

For the given values, RC over load,

$$\mathbf{\Gamma}_l = \frac{\mathbf{Z}_l - \mathbf{Z}_o}{\mathbf{Z}_l + \mathbf{Z}_o} = \frac{\left(12 + j24\right) - 60}{\left(12 + j24\right) + 60} = 0.707\angle 2.356$$

Magnitude of the coefficient, Γ_l and its phase, θ_l are 0.707 and 2.356 rad. Susceptance of the stub can be obtained as,

$$b = \pm \frac{2\Gamma_l}{\sqrt{1-\Gamma_l^2}} = \pm \frac{2\times 0.707}{\sqrt{1-0.707^2}} = \pm 2.00$$

Distance of stub from load can be found as,

$$d_s = \frac{\lambda}{4\pi}\left(2.356 \mp \frac{\pi}{2} \mp \tan^{-1}\frac{2.00}{2} - 2n\pi\right)$$
$$= \lambda\left(0.187 \mp 0.125 \mp 0.0625 - 0.5n\right)$$
$$= \begin{cases} \left(-0.0005 - 0.5n\right)\lambda & \text{for } b > 0 \\ \left(0.3745 - 0.5n\right)\lambda & \text{for } b < 0 \end{cases}$$

In the above expressions, appropriate value for n is −1 for b>0 and 0 for b<0. Thus, distance of stub from load is,

$$d_s = \begin{cases} 0.4995\lambda & \text{for } b > 0 \\ 0.3745 & \text{for } b < 0 \end{cases}$$

The length of the stub, for each case can be found as

$$l_s = \begin{cases} \frac{\lambda}{2\pi}\left(\pi - \tan^{-1}\frac{1}{2}\right) = \frac{\lambda}{2\pi}(\pi - 0.464) = 0.426\lambda & \text{for } b > 0 \\ \frac{\lambda}{2\pi}\left(\tan^{-1}\frac{1}{2}\right) = \frac{\lambda}{2\pi}(0.464) = 0.0738\lambda & \text{for } b < 0 \end{cases}$$

Closest postion and corresponding length of stub are, 0.3745λ and 0.0738λ.

Example 15.17: A 50Ω loss-less line is to be matched to a complex impedance load by means of a short circuited stub. Given that stubs are made from the main line and complex impedance load $\mathbf{Z}_l$= (30– *j*40)Ω. Determine the stub position closest to the load and its length so that match is obtained.

***Solution*:**

Given that $\mathbf{Z}_o$ = 50Ω and $\mathbf{Z}_l$ = (30– *j*40)Ω. Substituting the given values, one can obtain, the RC over load as,

$$\mathbf{\Gamma}_l = \frac{\mathbf{Z}_l - \mathbf{Z}_o}{\mathbf{Z}_l + \mathbf{Z}_o} = \frac{30 - j40 - 50}{30 - j40 + 50} = 0.5\angle -1.57$$

Magnitude of the coefficient, Γ_l and its phase, θ_l are 0.5 and −1.57 rad. Susceptance of the stub

$$b = \pm\frac{2\Gamma_l}{\sqrt{1-\Gamma_l^2}} = \pm\frac{2\times 0.5}{\sqrt{1-0.5^2}} = \pm 1.155$$

Distance of stub from load is,

$$d_s = \frac{\lambda}{4\pi}\left(-1.57 \mp \frac{\pi}{2} \mp \tan^{-1}\frac{1.154}{2} - 2n\pi\right)$$

$$= \begin{cases} \frac{\lambda}{4\pi}(-\pi - 0.523 - 2n\pi) & \text{for } b > 0 \\ \frac{\lambda}{4\pi}(0.523 - 2n\pi) & \text{for } b < 0 \end{cases}$$

$$= \begin{cases} (-0.292 - 0.5n)\lambda & \text{for } b > 0 \\ (0.042 - 0.5n)\lambda & \text{for } b < 0 \end{cases}$$

In the above, the values for *n* are −1 for *b* > 0 and zero for *b* < 0, resulting in a distance of less than one half wavelength.

$$d_s = \begin{cases} (0.208)\lambda & \text{for } b > 0 \\ (0.042)\lambda & \text{for } b < 0 \end{cases}$$

The length of the stub for such case is

$$l_s = \begin{cases} \frac{\lambda}{2\pi}\left(\pi - \tan^{-1}\frac{1}{1.154}\right) = \frac{\lambda}{2\pi}(\pi - 0.714) = 0.386\lambda & \text{for } b > 0 \\ \frac{\lambda}{2\pi}\left(\tan^{-1}\frac{1}{1.154}\right) = \frac{\lambda}{2\pi}(0.714) = 0.1136\lambda & \text{for } b < 0 \end{cases}$$

Closest postion and corresponding length of stub are given by 0.041λ and 0.1136λ.

Example 15.18: A 50Ω transmission line, working at 0.5 GHz, having a wave velocity, v=1.5×10^8 m/s is terminated on an unknown impedance. It is found that, VSWR is 4 and the first minimum is formed at 2cm from the load end. Design a single stub impedance matching for the given conditions.

***Solution*:**

The wavelength corresponding 1 GHz frequency, assuming wave velocity equal to free space velocity, is $\lambda = v/f$=1.5×10^{10}/0.5×10^9=30cm. To find the distance and length of the stub, it requires the magnitude of RC and angle of RC over the load. These two parameters can be obtained from the given data as follows.

$$\Gamma_l = \frac{\rho - 1}{\rho + 1} = \frac{4-1}{4+1} = 0.6$$

$$\theta_l - 2\beta d_{\min} = -\pi \rightarrow \theta_l = 2\beta d_{\min} - \pi$$

$$\theta_l = 2 \times \frac{2\pi}{\lambda} \times 2 - \pi = 2 \times \frac{2\pi}{30} \times 2 - \pi = -0.73\pi \text{ rad}$$

Magnitude of the coefficient, Γ_l and its phase, θ_l are 0.6 and −0.73π rad. Now, the susceptance of stub can be obtained as,

$$b = \pm \frac{2\Gamma_l}{\sqrt{1-\Gamma_l^2}} = \pm \frac{2 \times 0.6}{\sqrt{1-0.6^2}} = \pm \frac{1.2}{0.8} = \pm 1.5$$

Distance of stub from load can be found as,

$$d_s = \frac{\lambda}{4\pi}\left(-0.73\pi \mp \frac{\pi}{2} \mp \tan^{-1}\frac{1.5}{2} - 2n\pi\right)$$

$$= \begin{cases} \frac{\lambda}{4\pi}\left(-1.23\pi - 0.643 - 2n\pi\right) & \text{for } b > 0 \\ \frac{\lambda}{4\pi}\left(-0.23\pi + 0.643 - 2n\pi\right) & \text{for } b < 0 \end{cases}$$

$$= \begin{cases} \lambda\left(-0.359 - 0.5n\right) & \text{for } b > 0 \\ \lambda\left(-0.006 - 0.5n\right) & \text{for } b < 0 \end{cases}$$

In the above, appropriate value value for n is –1.

$$d_s = \begin{cases} 0.141\lambda = 4.23\text{cm} & \text{for } b > 0 \\ 0.494\lambda = 14.82\text{cm} & \text{for } b < 0 \end{cases}$$

Now, the length of the stub can be computed as,

$$l_s = \begin{cases} \frac{\lambda}{2\pi}\left(\pi - \tan^{-1}\frac{1}{1.5}\right) & \text{for } b > 0 \\ \frac{\lambda}{2\pi}\left(\tan^{-1}\frac{1}{1.5}\right) & \text{for } b < 0 \end{cases}$$

$$= \begin{cases} \frac{\lambda}{2\pi}\left(\pi - 0.588\right) = 12.19\text{cm} & \text{for } b > 0 \\ \frac{\lambda}{2\pi}\left(0.588\right) = 2.81\text{cm} & \text{for } b < 0 \end{cases}$$

Closest postion and corresponding length of stub are given by 4.23cm and 12.19cm.

15.2.3. Double stub matching

In certain situations, single stub matching is difficult to implement, particularly in case of coaxial lines, where it is just not possible to place the stub physically over the line in an ideal location. In double stub matching, two stubs are used whose lengths can be adjusted at will, giving more degrees of freedom to the matching designer. The stubs can be connected as near to load as possible and the lengths of the stubs are adjusted to get proper matching, as shown in Figure 15.10.

Now, analytical expressions for susceptances, and from them, the lengths of stubs to be connected to the line for matching purpose, are derived. Here, the inter-stub spacing is designated by d_{ss}, distance of the near-stub from load by d_n , the length of near-stub by l_{sn} and the length of the far-stub by l_{sf} . After connection of the far-stub, the normalized admittance at its point of connection must become unity, to achieve matching. If the far-stub susceptance is b_f , then earlier to its connection, the admittance, $\mathbf{y}'_f$ and RC, $\mathbf{\Gamma}'_f$ of the line at the point of its connection are,

$$\mathbf{y}'_f = 1 - jb_f \qquad (15.15)$$

$$\mathbf{\Gamma}'_f = \frac{1-\mathbf{y}'_f}{1+\mathbf{y}'_f} = \frac{jb_f}{2-jb_f} \qquad (15.16)$$

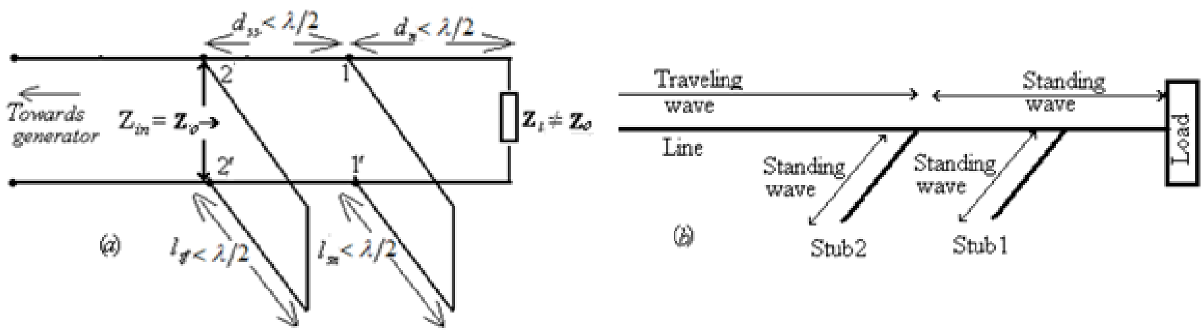


Figure 15.10 Load matching with shorted double stubs. (a)Stub connected at 11' is near stub and the one at 22' is far stub and (b) type of waves over various parts of the system.(a=13.4modified)

Then, the RC of the line at the location of the near-stub becomes,

$$\mathbf{\Gamma}_n = \mathbf{\Gamma}'_f e^{j\beta d_{ss}} = \frac{jb_f}{2-jb_f} e^{j\beta d_{ss}} \qquad (15.17)$$

The normalized admittance of the line at the point of near-stub location is,

$$\mathbf{y}_n = \frac{1-\mathbf{\Gamma}_n}{1+\mathbf{\Gamma}_n} = \frac{4 - j\left(4b_f \cos 2\beta d_{ss} - 2b_f^2 \sin 2\beta d_{ss}\right)}{4 - 4b_f \sin 2\beta d_{ss} + 4b_f^2 \sin^2 \beta d_{ss}} \qquad (15.18)$$

If the near-stub susceptance is b_n, then before its connection, the admittance of the line at the point of its connection is,

$$\mathbf{y}'_n = \mathbf{y}_n - jb_n = \frac{1}{1 - b_f \sin 2\beta d_{ss} + b_f^2 \sin^2 \beta d_{ss}}$$

$$+j\left(\frac{b_f^2\sin 2\beta d_{ss}-2b_f\cos 2\beta d_{ss}}{2-2b_f\sin 2\beta d_{ss}+2b_f^2\sin^2\beta d_{ss}}-b_n\right)$$
$$=g_n'+jb_n' \qquad (15.19)$$

Equating the real parts of both sides results in

$$\frac{1}{1-b_f\sin 2\beta d_{ss}+b_f^2\sin^2\beta d_{ss}}=g_n' \qquad (15.20)$$

Solving this, Eq. (15.20) for b_f results in

$$b_f=\frac{\sin 2\beta d_{ss}\pm\sqrt{\sin^2 2\beta d_{ss}-4\left(1-1/g_n'\right)\sin^2 2\beta d_{ss}}}{2\sin^2\beta d_{ss}}$$
$$=\frac{\cos\beta d_{ss}\pm\sqrt{1/g_n'-\sin^2\beta d_{ss}}}{\sin\beta d_{ss}} \qquad (15.21)$$

Double stub matching:

1. For double stub matching to be possible, the condition to be satisfied is $g_n'\le 1/\sin^2\beta d_{ss}$.
2. Other wise increase d_{ss} by $\lambda/4$so that the above condition is satisfied.

One can notice that the Eq. (15.21) cannot have a solution for b_f when value of $1/\sin^2\beta d_{ss}$ is less than g'_n. Hence, it is not possible to arrange matching for loads which results in such kind of inequality. A simple technique by which this difficulty can be overcome is to increase the inter stub spacing by a one quarter wavelength. However, if the value of $1/\sin^2\beta d_{ss}$ is more than or equal to g'_n, then b_f has two possible values. The corresponding values of b_n are, then given by,

$$b_n=\left(\frac{b_f^2\sin 2\beta d_{ss}-2b_f\cos 2\beta d_{ss}}{2-2b_f\sin 2\beta d_{ss}+2b_f^2\sin^2\beta d_{ss}}-b_n'\right) \qquad (15.22)$$

Once b_n and b_f are known, the lengths of the two stubs can be computed easily, as done in case of single stub matching technique.

Example 15.19: For each set of the given values, (a) $d_n = 0$, $d_{ss} = 3\lambda/8$ with $\mathbf{z}_l$=0.3+j0.4, (b) $d_n = \lambda/8$, $d_{ss} = 3\lambda/8$ with $\mathbf{z}_l$=0.5 and (c) $d_n = \lambda/4$, $d_{ss} = 5\lambda/8$ with $\mathbf{z}_l$=2.5–j5.0, determine whether double stub matching technique is feasible or not.

Solution:

It is feasible to achieve match with double stubs only when real part of input admittance of the line section, from stub 1 location to the load, is less than or equal to $1/\sin^2\beta d_{ss}$.

(a)Given, $d_n = 0$, $d_{ss} = 3\lambda/8$ with $\mathbf{z}_l$=0.3+j0.4. With $d_n = 0$, the tan βd_n becomes zero. It gives

$$\mathbf{y}_{in}=\frac{\left(1+j\mathbf{z}_l\tan\beta d_n\right)}{\left(\mathbf{z}_l+j\tan\beta d_n\right)}=\frac{1}{\mathbf{z}_l}=\frac{1}{0.3+j0.4}=1.2-j1.6$$

The real part of the admittance then is Re[$\mathbf{y}_{in}$] **=**1.2**.** Next, one can calculate the value of $1/\sin^2\beta d_{ss}$.

$$\beta d_{ss}=\frac{2\pi}{\lambda}\times\frac{3\lambda}{8}=\frac{3\pi}{4}=0.75\pi\rightarrow\frac{1}{\sin^2\beta d_{ss}}=\frac{1}{\sin^2 0.75\pi}=2$$

From the obtained values, as 1.2 < 2 i.e. Re[$\mathbf{y}_{in}$] <$1/\sin^2\beta d_{ss}$, double stub matching technique is possible.

(b)Given, $d_n = \lambda/8$, $d_{ss} = 3\lambda/8$ with $\mathbf{z}_l$=0.5. With $d_n = \lambda/8$, then

$$\tan\beta d_n = \tan\frac{2\pi}{\lambda}\times\frac{\lambda}{8} = \tan\frac{\pi}{4} = 1.$$

It gives,

$$\mathbf{y}_{in} = \frac{(1 + j\mathbf{z}_l\tan\beta d_n)}{(\mathbf{z}_l + j\tan\beta d_n)} = \frac{(1 + j0.5)}{(0.5 + j)} = 0.8 - j0.6$$

The real part of the admittance then is Re[$\mathbf{y}_{in}$] =0.8**.** Next, one can calculate the value of $1/\sin^2\beta d_{ss}$.

$$\beta d_{ss} = \frac{2\pi}{\lambda}\times\frac{3\lambda}{8} = \frac{3\pi}{4} = 0.75\pi \rightarrow \frac{1}{\sin^2\beta d_{ss}} = \frac{1}{\sin^2 0.75\pi} = 2$$

From the obtained values, as 0.8 < 2 i.e. Re[$\mathbf{y}_{in}$] <$1/\sin^2\beta d_{ss}$, double stub matching technique is possible.

(c)Given, $d_n = \lambda/4$, $d_{ss} = 5\lambda/8$ with $\mathbf{z}_l$=2.5–j5.0. With $d_n = \lambda/4$, then

$$\tan\beta d_n = \tan\frac{2\pi}{\lambda}\times\frac{\lambda}{4} = \tan\frac{\pi}{2} \rightarrow \infty.$$

It gives,

$$\mathbf{y}_{in} = \frac{(1 + j\mathbf{z}_l\tan\beta d_n)}{(\mathbf{z}_l + j\tan\beta d_n)} = \frac{\left[(1/\tan\beta d_n)1 + j\mathbf{z}_l\right]}{\left[(\mathbf{z}_l/\tan\beta d_n) + j\right]}$$

With $\tan\beta d_n \rightarrow \infty$, the normalized input admittance becomes

$$\mathbf{y}_{in} = \frac{(0 + j\mathbf{z}_l)}{(j)} = \mathbf{z}_l = 2.5 - j5.0$$

The real part of the admittance then is Re[$\mathbf{y}_{in}$] =2.5**.** Next, one can calculate the value of $1/\sin^2\beta d_{ss}$.

$$\beta d_{ss} = \frac{2\pi}{\lambda}\times\frac{5\lambda}{8} = \frac{5\pi}{4} = 1.25\pi \rightarrow \frac{1}{\sin^2\beta d_{ss}} = \frac{1}{\sin^2 1.25\pi} = 2$$

From the obtained values, as 2.5 > 2 i.e. Re[$\mathbf{y}_{in}$] >$1/\sin^2\beta d_{ss}$, double stub matching technique can not be applied.

15.3. SMITH CHART

Smith chart is the best known and widely used graphical aid in solving transmission line problems. It was developed in 1939 by Phillip Hagar Smith(1905–1987), an electrical engineer. He obtained BS degree in 1928 from Tufts College and when working for Bell Labs, he developed a polar plot of complex RC with the normalized impedance or admittance in a unity circle, the now well-known as Smith chart. The real utility of this chart is that it can be used to convert RC to its corresponding normalized impedance and admittances.

15.3.1 Smith chart features

The Chart is circular in shape, and it has two sets of curves: one set is of complete circles and the other one consists of circular arcs. They are described in a little more detail below:

1. The complete circles are called *constant resistance r circles* or *constant conductance g circles*. The centers of these circles lie over a horizontal straight line, passing through

center of the chart. These circles represent the normalized resistance/conductance at various points over a loss-less transmission line of one half wavelength long.

2. There is a horizontal straight line, dividing the chart into two, upper half and lower half, and passing through center of the chart. The center point is designated as '1', on the left side with values less than one, upto zero and on the right side with values more than one, upto infinity.
3. The circular arcs are called *constant reactance x arcs* or *constant susceptance b arcs*. These arcs lie on both sides of the horizontal line. They represent values of the normalized reactance/susceptance at various points over a loss-less transmission line of one half wavelength long. The horizontal line denotes zero reactance and susceptance.
4. The two sets of curves, one set of complete circles and one set of circular arcs, form two different families, and they are mutually orthogonal i.e. the circles and arcs are always orthogonal to each other. Also note that all the curves of both the sets, pass through the rightmost point of the chart, representing, Γ_r =1, Γ_i = 0.
5. The chart can be used either as an impedance chart or as an admittance chart. When used as impedance chart, the circles denote resistance and arcs represent reactance. However, the same circles denote conductance and same arcs represent susceptance, when it is used as admittance chart.
6. Movement from left to right, over the chart, corresponds to circles of decreasing radii, denoting the increasing in resistance/conductance. The largest circle at the border of chart denotes a resistance/conductance of zero.
7. The horizontal line divides the chart into an upper half and a lower half. The upper half of the chart denotes positive reactance/susceptance i.e. normalized inductive reactance/capacitive susceptance whereas the lower half represents negative reactance/susceptance i.e. the normalized capacitive reactance/ inductive susceptance.
8. The chart can also be divided as left half and right half. In the left half, resistance/conductace values and reactance/susceptanace values are less than one. In the right half, resistance/conductace values and reactance/susceptanace values are more than one
9. The chart can represent the line impedance only for *one half-wavelength* long. However, as the line characteristics are periodic with a periodicity of one half-wavelength, the chart can be made use for any length of line.
10. The chart can be used only with *normalized* quantities i.e. normalized impedances/ admittances. The normalization is with respect to the load impedance.
11. In the upper half of chart, clockwise movement corresponds to increase in inductive reactance/capasitive susceptance whereas in the lower half, clockwise movement corresponds to decrease in capacitive reactance/inductive susceptance.
12. The movement, on the line, towards the generator corresponds to clockwise motion on the chart and towards the load corresponds to anti-clockwise motion.
13. The horizontal radius to the right of the chart centre corresponds to voltage maxima, $V_{\max}$ current minima, $I_{\min}$ impedance maxima, $z_{\max}$ and SWR, ρ and left of the chart centre corresponds to voltage minima, $V_{\min}$ current maxima, $I_{\max}$ impedance minima, $z_{\min}$ and inverse SWR, $1/\rho$. Also the location of $V_{\max}$ corresponds to $z_{\max} = \rho$ on the line and that of the $V_{\min}$ corresponds to a point of $z_{\min} = 1/\rho$.

14. Radial lines represent loci of the constant line angle, βz. In the chart, wavelength scales corresponding to the line angle are included around the outside edge of the chart.
15. For a lossy line not terminated in its characteristic impedance the path of travel on the chart from the load to the generator is a decreasing logarithmic spiral.

Theory: The normalized impedance, **z** and RC, **Γ** at any arbitrary point over a loss-less line are related through,

$$\mathbf{z} = \frac{1+\mathbf{\Gamma}}{1-\mathbf{\Gamma}} \qquad (15.23)$$

Both the impedance and RC are complex, having real and imaginary parts. Let $\mathbf{z} = r + jx$ and $\mathbf{\Gamma} = \Gamma_r + j\Gamma_i$. Then the above relation can be written as,

$$r + jx = \frac{1+\Gamma_r + j\Gamma_i}{1-\Gamma_r - j\Gamma_i}$$

After multiplying the numerator and denominator of the right hand side of the above equation with conjugate of denominator, equating real parts on both sides of the equation results in,

$$r = \frac{1-\Gamma_r^2-\Gamma_i^2}{(1-\Gamma_r)^2+\Gamma_i^2} \qquad (15.24a)$$

And, equating imaginary parts on both sides of the equation results in,

$$x = \frac{2\Gamma_i}{(1-\Gamma_r)^2+\Gamma_i^2} \qquad (15.24b)$$

These two equations can be rearranged as

$$\left(\Gamma_r - \frac{r}{1+r}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r}\right)^2 \qquad (15.25a)$$

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2 \qquad (15.25b)$$

16. One can notice that these two expressions, Eqs. (15.25a&b), represent circles on complex RC plane. The first one represents a family of constant resistance circles with radius $1/(1+r)$ and centre at $r/(1+r)$ along the real axis. The second one represents a family of constant reactance circles with radius $1/x$ and centre at $\Gamma_r=1$, $\Gamma_i=1/x$. The salient features of the Smith chart are illustrated in Figure 15.11.

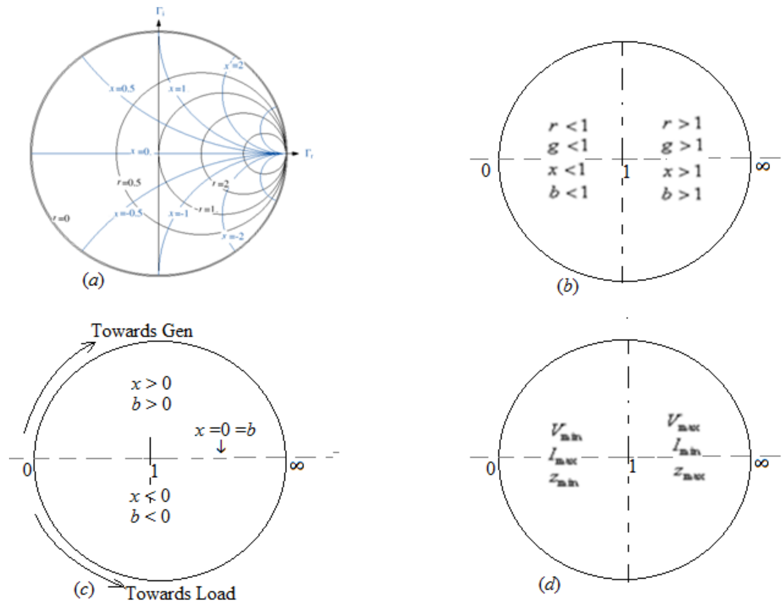


Figure 15.11 Illustrating formation and salient features of Smith chart.(13.5and 13.7modified)

A chart that is used in practice is given in Appendix. To make use of the chart effectively, all its features are required to be known. One among them is about 'constant SWR circle' which is elaborated below:

1. The 'constant SWR circle', also known as 'constant ρ circle', is a circle drawn on the chart with SWR as radius and '1' of the horizontal line as centre.
2. The impedances represented by various points over the SWR circle denote the impedance of line at various points within a length of $\lambda/2$.
3. Distance along the line is represented by angular distance around the chart, total circumference or 360^0 corresponding to the line length of $\lambda/2$.
4. The point at which the constant SWR circle intersects the horizontal straight line, right side of center, corresponds to SWR of the load.
5. For each different value of SWR, there exists a separate circle with a different radius.
6. Constant SWR circles are concentric circles, with centres over '1', having different radii representing different SWR values. These circles intersect horizontal line at points,$1/\rho$ and ρ, representing impedances equal to Z_o/ρ and $Z_o\rho$where Z_o is characteristic impeance

15.3.2. Smith Chart Utility

Smith chart can be used to find a variety of quantities of transmission lines without doing complex computations. Some of them are mentioned below.

• VSWR over the line: To find VSWR over the line, first normalize the load impedance with characteristic impedance, Z_o of the line and let it be $r + jx$. Now locate this impedance over the Smith chart at a point, where circle of r and arc of x intersect and let it be at P, as shown in Figure 15.10(a). Now, draw VSWR circle, with center, O and radius equal to distance between P and O, O being center of the Chart. This circle cuts 1-∞ segment of horizontal line of chart, and let it be Q. Identify the circle that passes through this point Q and its r value gives required VSWR.

• Location of voltage maximum and minimum: First of all, normalize the load impedance with Z_o of the line and locate this impedance over the Chart at a point *P*(say). Draw a radial vector joining center of chart *O* with point *P*. Next measure angles made by this vector, clockwise, with 1-∞ segment and 1-0 segment of horizontal line of the Chart, θ_1 and θ_2 as shown in Figure 15.10(b). These angles when divided by β give distance, in mts, of first maximum and first minimum from the load.

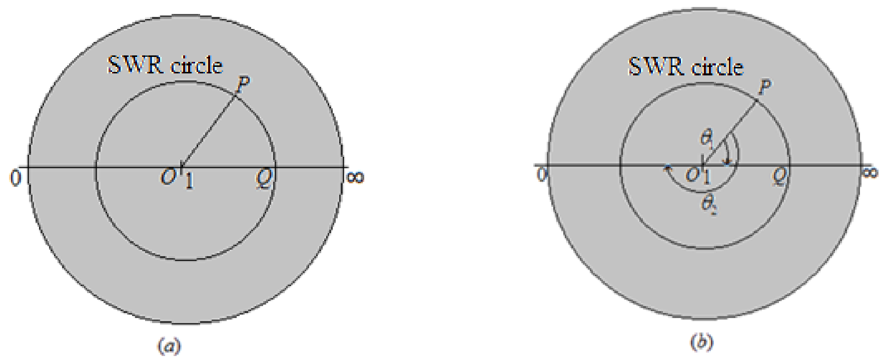


Figure 15.12 SWR, location of first mximum and minimum.

• RC over load: To find RC over the load, first normalize the load impedance with Z_o of the line and let it be $r + jx$. Now locate this impedance over the Smith chart at a point, let us say, at *P*. The distance of this point *P* to center of chart *O* gives magnitude of RC. To find angle of coefficient, draw a radial vector, from *O* to *P* and the angle made by this vector, clockwise, with 1-0 segment of horizontal line of chart gives required angle, θ_1 as shown in Figure 15.11(a).

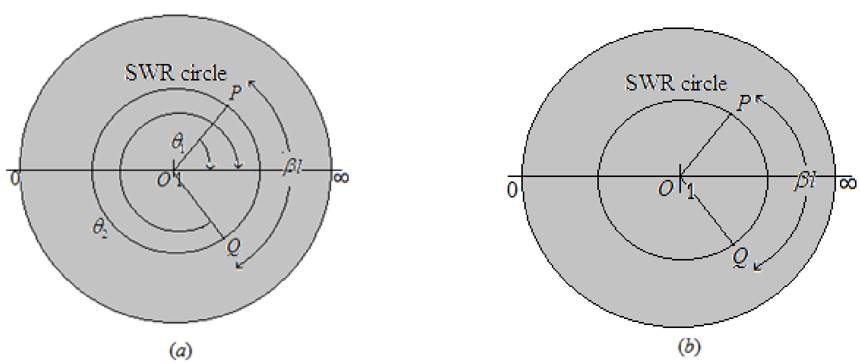


Figure 15.13 RC and impedance computation.

• RC at a distance from load: Let us suppose, RC is required at a distance of *l* from load. First normalize the load and locate the normalized load over the Smith chart, let us say at point, *P*. Now draw constant SWR circle passing through *P* and rotate the radial vector *OP* clockwise an angle, βl moving the load point to another location, *Q* on the circle.
The distance of this point *Q* to center of chart *O* gives magnitude and the angle, clockwise, made by radial vector, from *O* to *Q*, with 1-∞ segment of horizontal line of chart gives angle of RC at a distance *l* from load, θ_2 as shown in Figure 15.11(a).

• Impedance at a distance from load: Let us suppose, impedance required is at a distance of *l* from load. First, locate the normalized load over the Smith chart, let us say at a point, *P*. Now

draw constant SWR circle passing through P and rotate the radial vector OP clockwise an angle, βl moving the load point to another location, Q on the circle.
Now, identify r and x values of circles/arcs passing through point Q, which gives the normalized impedance over the line at a distance of l form load. By multiplying normalized impedance with Z_o of line then gives the required impedance in ohms. This process can also be used to identify the type of load at any point over the load.

15.3.3. Single Stub Matching

The location and length of the stub to be placed over the line for matching purpose can be calculated using Smith chart. The procedural steps to be followed are given hereunder.

- Plot the normalized impedance and draw the constant SWR circle on Smith chart.
- Move a distance of λ/4 along the constant SWR circle to locate load admittance. Let it be P_1.
- On the SWR circle nearest to the load admittance point locate a point, which represents admittance $1\pm jb$. This is the point of intersection of constant SWR circle and r =1 circle. Let it be P_2.
- Read the distance between P_1 and P_2 using the scale provided at the circumference of the chart. This gives the distance in wavelengths where the stub has to be placed from the load.
- Starting from the point, $(\infty, j\infty)$, find the distance of the point at which the susceptance is $\pm jb$. This gives the length of the short-circuited stub in wavelengths to be connected for matching.

15.3.4 Double-Stub Matching

Single stub matching is impractical when the stub is not being able to be placed physically in the ideal location. Particularly, in case of coaxial lines, it is very difficult to place the stub at the exact required location.

In double stub matching, the stubs are connected as near to load as possible and the lengths of the stubs are adjusted to get proper matching. For the sake of convenience, let us call the stub that is nearest to the load as near-stub and the other one as far-stub. The spacing between the stubs is usually either λ/8 or 3λ/8 or 5λ/8 but most often it is λ/4. The spacings of λ/2 is avoided, because it places the two stubs in parallel resulting in the availability of only single effective adjustment. The close spacing of the stubs is also avoided for the same reason. Now, the elements of double stub matching with Smith chart are briefly given.

Spacing circle: It is a circle obtained by rotating the constant conductance circle g = 1 around the center of the Smith chart. The amount of rotation depends upon the spacing of the stubs. For a spacing of either λ/8 or 5λ/8, the amount of rotation is 90^0 anti-clockwise and in case of 3λ/8, the rotation is 90^0 clockwise. If the spacing happens to be λ/4 then the amount of rotation is 180^0.

To understand the significance of spacing circle, the line in between the stubs has to be viewed as a transformer, converting the admittances at far-stub location into different admittance values at near-stub location. The admittance of the line at far-stub location is $1\pm jb$ and, therefore, the constant conductance circle, g = 1describes these admittances. The admittances the location of near-stub then can be sketched over the Smith chart by moving the admittance points along constant SWR circles by angle corresponding the length of the transformer. If all the admittance points at the location of stub 2, are joined, it also results in the so called 'spacing circle'.

Forbidden region: The double stub matching technique does not work for certain values of load admittances. The region of the Smith chart that represents all these values of load admittances which are not amenable for this technique is called 'forbidden region'. The region basically depends upon the spacing of the stubs. When the first stub is right over the load, and for a stub spacing of either λ/8 or 3λ/8 or 5λ/8 the forbidden region consists of the entire area encircled by constant conductance g = 2 circle. If it is equal to λ/4, then the forbidden region is the entire area encircled by constant conductance g = 1 circle.

Location of stubs: For reflection-less operation of line, the input admittance of line looking towards the load at far-stub location shall equal to the characteristic impedance. Or at the location of the far-stub the real part of the admittance of the line is required to be one. The far-stub is used to cancel the imaginary part of the line admittance at its location resulting in normalized admittance of one.

It is always desirable for the near-stub to be connected at or near to the load. But it is not always possible to have such a connection because of the existence of forbidden regions.

Functioning: Stub 1 transforms the input admittance of the line and load to the right of its location into an admittance which lies over the spacing circle. The line transformer further transforms this admittance into a value which lies over unity conductance circle. The susceptance of the line at this point is tuned out using stub 2.

Procedural steps:

Step I: Locate the normalized admittance over the chart. Let it be *P*.

Step II: From *P* move clockwise over constant SWR circle, a distance corresponding distance of the first stub from the load. Let the point be *Q*. If the first stub is to be placed right over the load, then this step becomes redundant.

Step III: From *Q* move over constant g circle until spacing circle is met. Let the point be *R*. In case of two possibilities i.e. clockwise and anti-clockwise either one can be selected. From the susceptance value of the point *R*, the length of the first stub can be calculated.

Step IV: From *R* move over constant SWR circle until g = 1 circle is met. Let it be *S*. From the susceptance value of the point *S*, the length of the second stub can be calculated. Note that the SWR circle in the present step is different from that of step II.

Example 15.20: Using the Smith chart, determine VSWR ρ, the location of the first V_{max} and first V_{min} from the load, when the line is connected to a normalized impedance, equal to (2+j2)Ω and functioning at an operating wavelength of 6cm.

***Solution*:**

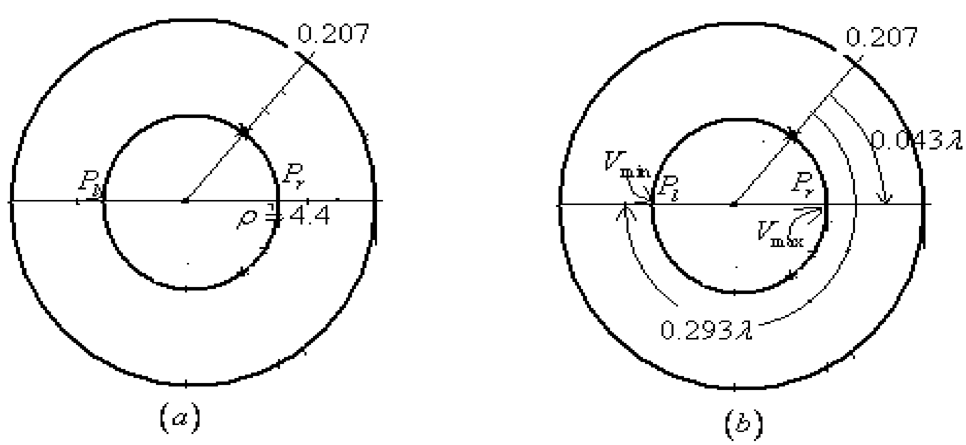


Figure 15.14 Illustrating Smith chart operations pertaining to Example 15.20. (13.9)

The solution procedure involves several steps which are explained here in a detailed manner, with reference to the Figure 15.14.

- The normalized impedance is given. Locate this impedance on a Smith chart and let it be designated as point *P*.

- With center of the chart as center and center to point *P* as radius, draw the constant SWR circle.
- The constant SWR circle cuts the horizontal line of the chart at two points, one on left of the center P_l and the other on the right of the center P_r. The point on the right of the center gives the SWR over the line. From the chart, it is 4.4.
- From point P read the angular distance in clockwise direction over the SWR circle to P_l and P_r and convert them into linear distances. The first one gives the distance of the first $V_{\min}$ and the second one gives the first $V_{\max}$ and from the load. From the chart they are 0.293λ and 0.043λ respectively.

Example 15.21: A 50 Ω transmission line is terminated over a load of impedance (100+*j*50)Ω. Using the Smith chart, determine (a)VSWR ρ,(b)RC,(c) distance of voltage minimum from the load, (d) line impedance at a distance of 0.15λ form the load(e) line admittance at a distance of 0.15λ form the load and (f) the location of the nearest point to the load where the real part of the line admittance is equal to the line characteristic admittance.

***Solution*:**

The solution procedure involves several steps, which are explained here in a detailed manner, with reference to the Figure 15.15.

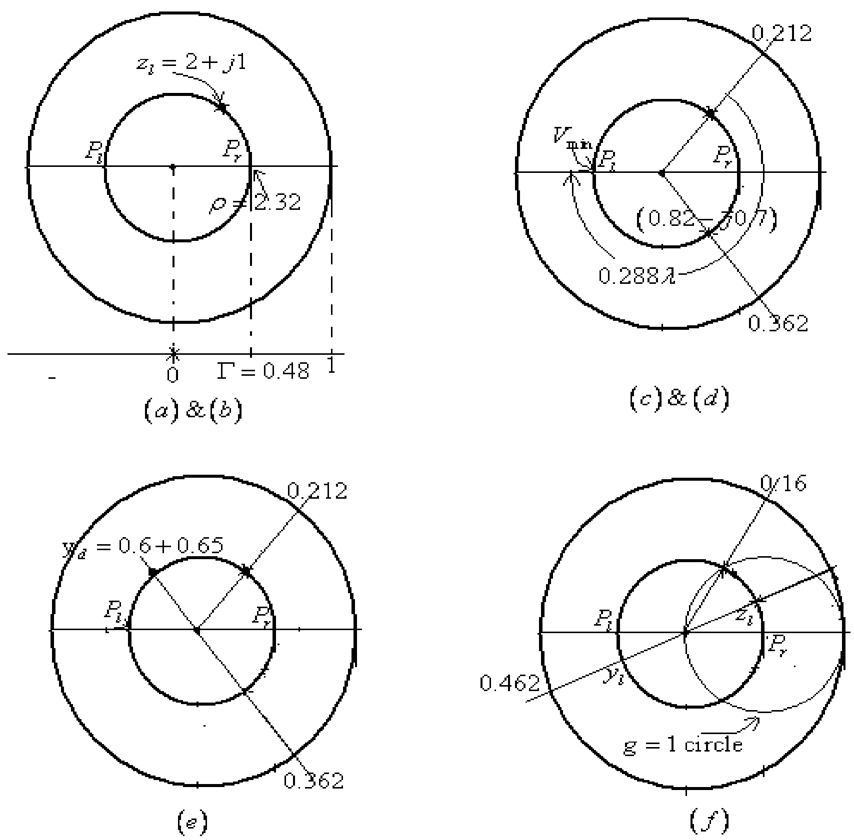


Figure 15.15 Illustrating Smith chart operations pertaining to Example 15.21.(13.8)

- The load impedance and characteristic impedances are given. The normalized load impedance is,

$$z_l = \frac{100 + j50}{50} = 2 + j1$$

- Locate the normalized impedance over Smith chart and with 1 as center and from 1 to the point representing the normalized impedance as radius, draw the constant SWR circle. The circle cuts the real or horizontal axis at two points. The value against the point right of the center is SWR. From the Smith chart operation, as shown in Figure 15.15(a), SWR is found equal to 2.32.
- From the horizontal scale provided over the bottom of the chart, RC can be computed. As shown in Figure 15.15(b), it is 0.48.
- The distance between the load point to the point of intersection of real axis and SWR circle on the left side of the center can be read using the scale provided over the periphery of the chart. The load point location is 0.212 and the voltage minimum point is 0.50. Therefore the distance of the first voltage minimum the load is 0.50–0.212=0.288 times wavelength. It is shown in Figure 15.15(c)
- The required line impedance can be found by moving a distance of 0.15λ over the SWR circle in clockwise direction. The point represents normalized impedance of $\mathbf{z}_d$=(0.82–j0.7). The absolute impedance, shown in Figure 15.15(d), then is $\mathbf{Z}_d$=50×(0.82–j0.7)= (41–j35).
- Locate the point over the SWR circle diametrically opposite to the point located in the previous step. It represents the required admittance. Its normalized value from the chart is $\mathbf{y}_d$=(0.6+j6.5). The absolute admittance, shown in Figure 15.15(e), then is $\mathbf{Y}_d$=(0.6+j6.5)/50= (0.012+j0.13).
- The distance of the nearest point to the load where the real part of the line admittance is equal to the line characteristic admittance can be found by locating the load admittance over the SWR circle and then finding the distance in clockwise between this point and the point of intersection of SWR circle with g=1 circle. From the chart the load admittance point is 0.462 and the intersection point is 0.16. the distance between them then can be found to be 0.198 times wavelength. It is shown in Figure 15.15(f)

Example 15.22: Using the Smith chart, find the terminated impedance of a line of characteristic impedance of, $\mathbf{Z}_o$ =50Ω having an SWR=3. When the load is shorted, the shift in minima is 0.2λ towards the generator.

***Solution*:**

The solution procedure involves several steps which are described below, with reference to the Figure 15.16.

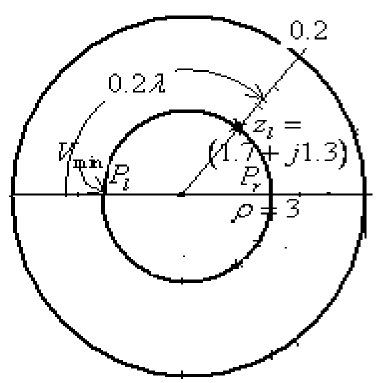


Figure 15.16 Illustrating Smith chart operations pertaining to Example 15.22. (13.10)

- Take a Smith chart and with center of the chart as center and center to 3 over the horizontal axis as radius draw the constant SWR circle.
- The constant SWR circle cuts the horizontal line of the chart at two points, one on left of the center P_l and the other on the right of the center P_r.

- The point P_l corresponds to V_{min} , and now move over the SWR circle from P_l a distance corresponding to 0.2λ towards the generator and locate the load point. It gives the normalized impedance and from the chart its value is $\mathbf{z}_l$=(1.7+*j*1.3). Multiplication with $\mathbf{Z}_o$ =50Ω gives the actual value of the impedance. It is $\mathbf{Z}_l$=50×(1.7+*j*1.3)= (85+*j*65).

Example 15.23: Using the Smith chart, find the location and length of the short circuited stub required for a line of $\mathbf{Z}_o$ =50Ω when the normalized load admittance is (2+*j*1.75)℧. The characteristic impedance of stub is 100Ω. .

***Solution*:**

The solution procedure is a multi-step process, described below with respect to the Figure 15.17.

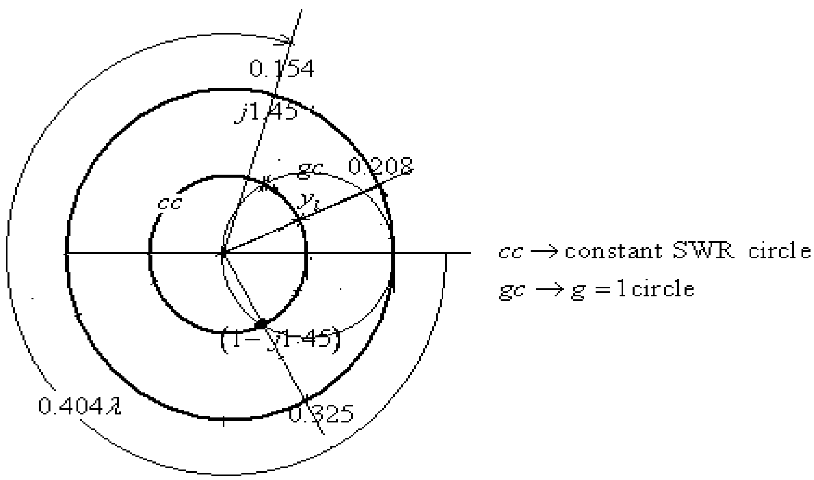


Figure 15.17 Illustrating Smith chart operations for single stub matching pertaining to Example 15.23. (13.11)

- The normalized admittance is given. Locate the normalized admittance on the Smith chart and then draw the constant SWR circle.
- On the SWR circle nearest to the load admittance point locate a point, in clockwise direction, which represents admittance 1± *jb*. This is the point of intersection of constant SWR circle and *g*=1circle. For the given data, from the Smith chart it is (1– *j*1.45)
- Read the distance between the points identified in the previous steps using the scale provided at the circumference of the chart. This gives the distance in wavelengths where the stub has to be placed from the load. From the chart it is 0.325–0.208=0.117 times wavelength
- Starting from the point (∞, *j*∞), find the distance of the point at which the susceptance is *j*1.45. This gives the length of the short-circuited stub in wavelengths to be connected for matching. From the chart it is 0.404λ.

Example 15.24: In a double stub matching scheme, the terminating impedance, $\mathbf{Z}_l$ is (100–*j*100)Ω and the characteristic impedances, $\mathbf{Z}_o$ of the line and the stub, both are equal to 50Ω. The first stub is placed at 0.05λ away from the load. The spacing between the two stubs is (3/8)λ. Determine the length of the short-circuited stubs, when the match is achieved.

Solution:

The solution procedure involves several step which are described below, with reference to the Figure 15.18.

- From the given values of the load impedance and characteristic impedance, find the normalized impedance. And then locate the normalized admittance over the chart. From the Smith chart, the normalized admittance is (0.24+*j*0.25).
- From the point located in the first step, move clockwise over constant SWR circle, a distance corresponding distance of the first stub from the load. It is 0.05λ in the given Example. The admittance of the point reached, from the Smith chart, is (0.32+*j*0.6).
- Draw the spacing circle of (3/8)λ by rotating the constant-conductance unity circle (*g*=1) through a phase angle of $2\beta d = 2\beta(3/8)\lambda = (3/2)\pi$ toward the load.
- From the point located in the previous step move over constant *g* circle until spacing circle is met. Now, two points are encountered, meeting this condition. These two points in general give two different sets of lengths of the stubs. Here, only one point i.e. (0.32–*j*0.25) is considered and solution lengths are found. The solution is similar when the other point is selected.
- The difference in the susceptance values of previous two points has to be nullified by the first stub. Here it is –*j*0.25–*j*0.65=–*j*0.85. The stub susceptance has to be *j*0.85. From the chart the corresponding length of the first stub can be calculated. It is 0.14λ.

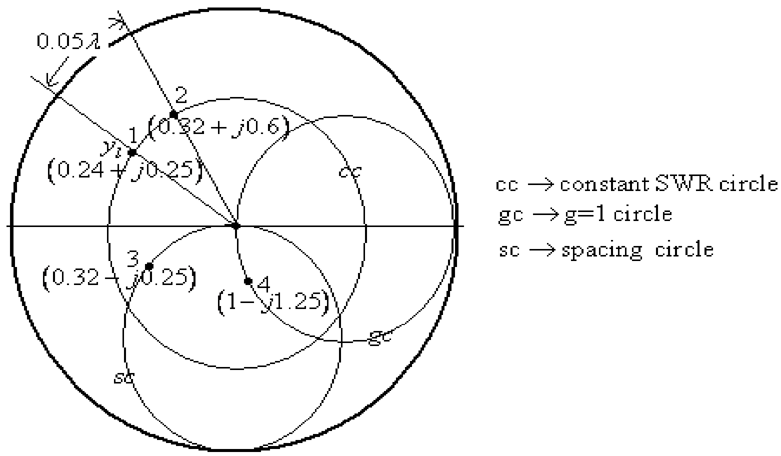


Figure 15.18 Illustrating Smith chart operations for double stub matching pertaining to Example 15.24.(13.12)

- From the point located in the previous step i.e. (0.32–*j*0.25) move over constant SWR circle and in clockwise direction until its corresponding point over (*g*=1)circle is met. From the susceptance value of this point, which is *j*1.25, the length of the second stub can be calculated. It is from the Smith chart 0.395λ. Note that the SWR circle in the present step is different from that of second step.

UNIT-III
TRANSMISSION LINE AT HIGHER FREQUENCIES
Assignment-Cum-Tutorial Questions

SECTION-A

1. The Smith chart can be characterized as [ ]
   a) A Polar plot b) represents complex RC
   c) Inscribed in a unity circle d) all
2. The complete circles and arcs in the Smith chart, respectively, represent [ ]
   a) Normalized resistance/conductance, Normalized reactance/ susceptance
   b)Normalized reactance/ susceptance, Normalized resistance/conductance
   c) Normalized reactance, susceptance
   d) none of these
3. The circles and arcs over the Smith chart are **[ ]**
   a) Orthogonal b) Opposite to each other
   c) At 45^0 d) None of these
4. The upper half and lower half of the Smith chart, respectively, represent [ ]
   a) Positive, nagative reactance/susceptances
   b) Capacitive, inductive reactance/susceptances
   c) Resistance, conductance d) None of these
5. The radius of the constant SWR circle is equal to [ ]
   a) Voltage SWR b) Current SWR
   c) Both (a) and (b) d) None of these
6. The centre of the constant SWR circle falls over **[ ]**
   a) '1'of horizontal line b) centre of the chart
   c) Both (a) and (b) d) None of these
7. In the left-half and right-half of the chart, resistance and reactance values, respectively, are [ ]
   a) More than 1, less than 1 b) Less than 1, more than 1
   c) 1,1 d) 0,0
8. The left most and rigtht most points of the chart, respectively, represent [ ]
   a) (0,0), (∞,∞) b) (∞,∞),(0,0),
   c) (0,0), (1,1) d) (1,1),(∞,∞)
9. The top most and bottom most points of the chart, respectively, represent [ ]
   a) (1,1), (–1,–1) b) (–1,–1), (1,1)
   c) (1,1), (0,0) d) None of these
10. Smith chart is always used with [ ]
    a) Normalized impedances b) Normalized admittances
    c) Both (a) and (b) d) None of these
11. The Smith chart is useful to analyze **[ ]**
    a) Loss-less lines b) lossy-lines
    c) Both (a) and (b) d) None of these
12. The horizontal line left and rigth of the centre, respectively, represent [ ]
    a) $\mathbf{V}_{max}$, $\mathbf{I}_{max}$; $\mathbf{V}_{min}$, $\mathbf{I}_{min}$ b) $\mathbf{V}_{min}$, $\mathbf{I}_{max}$; $\mathbf{V}_{max}$, $\mathbf{I}_{min}$
    c) $\mathbf{V}_{min}$, $\mathbf{I}_{min}$; $\mathbf{V}_{max}$, $\mathbf{I}_{max}$ d) None of these

13. The movement towards load and towards source over the line, respectively, correspond to, **[ ]**

a) Clock-wise, anticlock-wise rotation over the Smith chart
b) Anti-clockwise, clock-wise rotation over the Smith chart
c) Upwards, downwards d) None of these

14. Travel of length $\lambda_g/2$ over the line corresponds a rotation of [ ]

a) 180^0 over the chart b) 360^0 over the chart
c) 90^0 over the chart c) None of these

15. On the smith chart, full circles represents_________ [ ]

A) Inductance B) Capacitance C) Resistance D) Impedance

16. The radius of the constant SWR circle is equal to __________ [ ]

A) Reflection coefficient B) Transmission coefficient
C) SWR D) Normalized impedance

17. The centre of constant SWR circle falls over ________ [ ]

A) Centre of the chart B) '1' of horizontal line
C) both A and B D) None of these

18. The left most points of the smith chart represents__________ [ ]

A) Generator B) Load C) Short circuit impedance D) Open circuit impedance

19. Equation representing circles is ——— and that representing arcs of the chart is ———
20. Double stub matching is not possible when the load falls in——————————region.
21. Over the Smith chart, full circles represent————————and arcs represent————————
22. Over the Smith chart, the upper half is———————and lower half is——————————
23. Smith chart uses only—————— and describe the line for ——————————————
24. The centre of constant SWR circle is always the—————————————— of the chart.
25. V_{max}, I_{min} and ρ correspond to—————————— half the chart and I_{max}, V_{min} and $1/\rho$ correspond to———————————— half the chart.
26. In single stub matching, length of stub is ————— and location of stub is ——————
27. Define stub.
28. A stub can be shorted at one end. (yes/no)
29. If the VSWR=2, the magnitude of the reflection coefficient is [ ]

a) ¼ b) 1/5 c) 0 d) 1/3

SECTION-B

Descriptive questions

1. Explain construction of Smith chart. **[C03]**
2. Describe about the UHF lines at higher frequencies act as circuit elements. **[C02]**
3. Describe the method of single stub matching. Derive the relation for the length and location of the stub. **[C02]**
4. What is mean by load matching? Explain Quarter wave transform technique. **[C02]**

5. Define matched line. What are the advantages of transmission over matched line? Explain why a matched line does not carry reflected wave. **[C03]**
6. Explain the double stub matching. **[C02]**
7. Locate voltage max and min for pure resistive termination for a lossless transmission line. **[C02]**
8. Describe the procedure of load matching with quarter wave transformer for different types of loads. **[C02]**
9. Describe Smith chart and its salient features. **[C03]**

Problems

1. A loss-less transmission line with Z_0=100 Ω is 0.434 λ long and terminated at an impedance of 260+j180. Find (a) VSWR (b) Reflection Coefficient (c) Input impedance (d) Location of voltage maximum on the line. By using smith chart. Assume the line is placed in free space. **[C03]**

2. A load of 100+j150 Ω is connected to a 75 Ω (Z_0) lossless line. Find (a) VSWR (b) Reflection Coefficient (c) Input impedance at 0.4 λ from the load (d) the load admittance (e) Location of V_{max} and V_{min} with respect to the load if the line 0.4 λ long . **[C03]**

3. A lossless line of 300 Ω impedance is terminated with a load impedance of 100+j150 Ω. The frequency of operation is 60 MHz find the location of a single stub needed for impedance matching. **[C03]**

4. A load such as an antenna of impedance Z_L= 50-*j*100 Ω is connected to a lossless transmission line with characteristic impedance Z_0= 100 Ω. The line operates at 300 MHz and the speed of propagation on the line is 0.8*c. Find:
   i. The reflection coefficient at the load
   ii. The reflection coefficient at a distance of 20 m from load towards the generator
   iii. Input impedance at 20 m from the load
   iv. The standing wave ration on the line
   v. Locations of the first voltage maxima and the first voltage minima from the load.**[C03]**

SECTION-C

1. For pure reactance and pure resistance loads, load points over the Smith chart, respectively, stay at, [ ]

a) At the periphery, over the horizontal line b) Over the horizontal line, At the periphery
c) In the lower half, At the periphery d) In the upper half, over the horizontal line

2. For a match terminated loss-less line, the location of load point over the Smith chart is [ ]

a) At centre b) At periphery c) In the upper half d) In the lower half

Transmission lines and Waveguides(UNIT IV)

A. Questions testing the remembrance/understanding level of students

I. Objective/Multiple choice questions

1. The cut-off frequency of wave in between parallel plane conductors is ------
2. The cut-off wavelength of wave in between parallel plane conductors is ------
3. The phase shift constant for wave in between parallel plane conductors is ------
4. The phase shift constant for wave in between parallel plane conductors at cut-off is ------
5. The phase shift constant for wave in between parallel plane conductors at high frequencies is ------

6. Transmissions lines carry waves in------ mode where as waveguides carry in------ TEM mode.
7. Pure real value of propagation constant indicates------ attenuation and------ wave motion.
8. Pure imaginary value of propagation constant indicates------ attenuation and------ wave motion.
9. In TE wave the electric vector is------ transverse to the direction of propagation of wave.
10. In TE wave the magnetic vector has------ component along the direction of propagation of wave.
11. In TM wave the magnetic vector is entirely------ to the direction of propagation of wave.
12. In TM wave the electric vector has a------ along the direction of propagation of wave.
13. In TEM wave both the electric and magnetic vectors are entirely------ the direction of propagation of wave.
14. In mixed or hybrid wave, both the electric and the magnetic vector have components------ the direction of propagation of wave.

II. Descriptive questions

1. What is TE wave.
2. What is mode of wave.
3. What is principal wave.
4. Write cutoff frequency of wave in parallel plate transmission.

B. Questions testing the ability of students in applying the concepts

I. Multiple choice questions

1. Wave in guide travels through
(a) Guide walls (b) dielectric
(c) Both (a)and (b) (d) None
2. The propagation constant pure real implies
(a) Wave without attenuation (b) No wave motion
(c) Wave with attenuation (d) None
3. The propagation constant pure imaginary implies
(a) Wave without attenuation (b) No wave motion
(c) Wave with attenuation (d) None
4. The propagation constant complex implies
(a) Wave without attenuation (b) No wave motion
(c) Wave with attenuation (d) None

5. In *TM* wave, H can have component

a) Parallel to propagation b) Normal to propagation

c) Both (a) and (b) d) None of these

6. In *TE* wave, *H* can have component

a) Parallel to propagation b) Normal to propagation

c) Both (a) and (b) d) None of these

7. The lowest order *TE* wave in between parallel conducting plates is

(a) TE_{10} (b) Principal wave.

(c) Both(a) and (b). (d) None

8. The principal wave is

(a) TM_{00} (b) TEM wave

(c) Both(a) and (b). (d) None

9. The nature of the wave normal to plates is

(a) Pure standing (b) Pure traveling

(c) Impure traveling (d) None

10. The nature of the wave parallel to plates is

(a) Pure standing (b) Pure traveling

(c) Impure traveling (d) None

II. Problems

1. A parallel plate waveguide is having a dielectric medium with ε_r =2.25 and μ_r =1. Determine its spacing *a* when its dominant mode cutoff frequency is 5GHz.

Answers: 2cm

2. A parallel plate waveguide of spacing *a*= 4cm, is having a dielectric medium with ε_r =4 and μ_r =1. Determine the TE modes that can propagate when the frequency is 5GHz. Also find f_c and λ_g for each propagating mode.

Answers: *m*=1,2, f_c = 1.875,3.75GHz, λ_g =3.24,4.53cm

3. A parallel plate waveguide of spacing *a*= 5cm, is having free space medium in between. If it is excited with fundamental 2GHz and its harmonics, determine all the frequencies that propagate in TE_{10} mode.

Answers: 4, 6, 8GHz……etc.

4. In an air-dielectric parallel-plate waveguide of spacing *a*= 5cm, TE modes are excited with a field distribution at its mouth given by,

$$\mathbf{E} = 15\pi\hat{\mathbf{y}}\left(\sin 20\pi x + 0.35\sin 60\pi x\right)\sin 10^9 \pi t \ \text{ V/m}$$

5. Determine the propagating modes and deduce the expression for electric field of the propagating wave.

Answers: TE_{10} ; $\mathbf{E} = 15\pi\hat{\mathbf{y}}\sin 20\pi x \sin\left[10^9\pi t - \left(80\pi/3\right)z\right]$ V/m

6. A 4GHz wave is propagating in a nonmagnetic medium having a dielectric constant, ε_r =2.2. When the phase shift constant is found as 54^0 /cm, find the cutoff wave number.

Answers: 0.81rad/cm

C. Questions testing the Analyzing/evaluating/creative abilities of student

1. Prove that the infinite parallel plane conductors act as high pass filter. Define the terms cutoff frequency and cutoff wavelength.
2. What is the meaning of mode of the wave? Give a complete description of modal

propagation characteristics of waves in between infinite parallel plane conductors.

3. Define and differentiate phase velocity from group velocity.

4. What are two types of attenuations that exist in waveguides? Write down general expressions for both the types of attenuation.

D. Previous GATE/IES questions

1. Attenuation constant in Np/m due to conductor loss is

a) $\frac{\text{power dissipated/unit length}}{2\times\text{power flow down the guide}}$ b) $\frac{2\times\text{power flow down the guide}}{\text{power dissipated/unit length}}$

c) $\frac{\text{power dissipated/unit length}}{\text{power flow down the guide}}$ c) None of these

2. Reflective attenuation comes into being when the frequency of the wave is

a) Less than cut off frequency b) More than cut off frequency

c) Both (a) and (b) d) None of these

GUIDED WAVE ANALYSIS

Here, the field distribution in the region between two infinitely large parallel conducting plates, in the presence of traveling wave, is investigated. This topic is important for more than one reason. Parallel wire transmission lines, waveguides, and also coaxial lines are similar to parallel plate systems, having certain common structural features and, in fact, all the above mentioned electrical transmission systems can be derived from the parallel plate structure under consideration.

This plate system is capable of supporting both TEM and non-TEM waves. When the features that support TEM waves are accentuated, the structure becomes a parallel wire transmission line and when features that sustain non-TEM wave features are enhanced, it results in waveguides. Hence, analysis of wave guided by parallel plates give an indication of characteristics of TEM and non-TEM waves, apart being a necessary prerequisite know-how for the analysis of parallel wire transmission lines and waveguides. The methodology used, concepts applied and results obtained from the analysis of this configuration can be used for benefit in the study of, both, TEM wave carrying transmission lines as well as non-TEM wave carrying waveguides.

12.1.1. Fields in between conducting plates

In this section, the relations connecting transverse components with longitudinal ones are derived. Let us suppose the two conducting plates, infinite in extant, are parallel and one lying exactly over the *yz*- plane and another at a height of a from the bottom plate as shown in Figure 12.2. Let us also suppose that there exists a wave, traveling in the positive *z*- direction in between the plates. It is assumed that the plates are made with perfect conductors i.e. conductivity σ of the walls is ∞ and it is also assumed that hollow region is a perfect dielectric i.e. its conductivity, σ is zero.

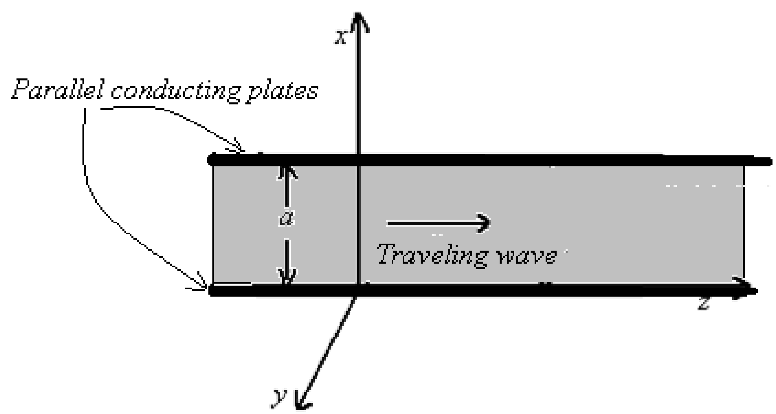


Figure 12.2 Parallel infinite conducting plates with traveling wave.(10.1 shaded)

The analysis is done in phasor domain i.e. time variations of the field quantities are assumed exponential i.e. $e^{j\omega t}$. When the time variations are exponential, the fields also vary in exponentially along the direction of the propagation, according to transmission line theory. So the fields must vary with z as $e^{\bar{\gamma} z}$ where $\bar{\gamma}$ is a constant, known as the propagation constant which, in general, is a complex quantity, given by, $\bar{\gamma} = \bar{\alpha} + j\bar{\beta}$.

Its real part, $\bar{\alpha}$ attenuation constant, represents the attenuation the wave undergoes while traveling and the imaginary part, $\bar{\beta}$, phase shift constant, denotes phase change in wave motion. In the ensuing analysis, loss free conditions are assumed and hence, it is un-attenuated wave transmission with a pure imaginary propagation constant, i.e. $\bar{\gamma} = j\bar{\beta}$. The 'bar' over the symbols indicate that they refer to non-TEM waves.

In general, the fields are function of x , y, z and t. In the present case, however, the plates are infinite in extant in y–direction. As there is no wave motion in that direction, the fields must be constant in the direction, and hence, independent of that dimension.

If **E** and **H** are electric and magnetic fields at an arbitrary point, P in the hollow region, they must be related through the Maxwell's curl equations, given by:

$$\nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E} \quad ; \quad \nabla \times \mathbf{E} = -j\omega\mu\mathbf{H} \tag{12.1}$$

In the analysis, it is aimed to find the fields **E** and **H,** which are vectors having all the three components. The expressions for all the six components, E_x , E_y , E_z , H_x , H_y and H_z of the fields are found, first by expressing the transverse components, E_x , E_y , H_x and H_y in terms of the longitudinal ones, E_z and H_z . Then the longitudinal components are found by solving their respective wave equations. The longitudinal fields, when expressed in terms of the transverse ones, appear as follows:

$$H_x = -\frac{\bar{\gamma}}{h^2}\frac{\partial H_z}{\partial x} \quad ; \quad H_y = -\frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x} \tag{12.2a}$$

$$E_x = -\frac{\bar{\gamma}}{h^2}\frac{\partial E_z}{\partial x} \quad ; \quad E_y = +\frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x} \tag{12.2b}$$

with

$$h^2 = \bar{\gamma}^2 + \omega^2\mu\varepsilon \tag{12.3}$$

Note that the relation in Eq. (12.3) is called *characteristic equation*. The constant, h also denoted by k_c , is called *cut-off wave number*. In the above relations, note that, all the field quantities are functions of x only.

Proof: The fields **E** and **H,** at point P obey Maxwell's curl equations. In phasor form equations, the fields are independent of time, t. If the z-variations of the field quantities are exponential, then, one can replace $\partial/\partial z$ by $-\bar{\gamma}$. In addition, as there is no wave motion in y-direction, fields must be uniform, resulting in $\partial/\partial y$=0. Expanding the first one of Eqs. (12.1) after considering these two aspects, results in,

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & 0 & -\bar{\gamma} \\ H_x & H_y & H_z \end{vmatrix} = j\omega\varepsilon\left(\hat{x}E_x + \hat{y}E_y + \hat{z}E_z\right) \tag{12.4}$$

Equating field components on both sides of Eq.(12.4), one can obtain,

$$\bar{\gamma}H_y = j\omega\varepsilon E_x \;; \frac{\partial H_z}{\partial x} + \bar{\gamma}H_x = -j\omega\varepsilon E_y \;; \frac{\partial H_y}{\partial x} = j\omega\varepsilon E_z \tag{12.5}$$

Similarly, by following same procedure, from the second one of the Eqs. (12.1), another set of three equations can be derived:

$$\bar{\gamma}E_y = -j\omega\mu H_x \,; \; \frac{\partial E_z}{\partial x} + \bar{\gamma}E_x = j\omega\mu H_y \,; \; \frac{\partial E_y}{\partial x} = -j\omega\mu H_z \tag{12.6}$$

After combining second of Eqs.(12.5) and first one of Eqs. (12.6) and then solving them for H_x results in an expression which is first one in Eq.(12.2a). In a similar manner, expressions for other transverse components can be computed by considering one equation from each set i.e., Eqs.(12.5) and (12.6).

Several critical observations can be made with respect to relations in Eqs.(12.3), which are listed below.

One: With $E_z = 0$, $H_z = 0$, simultaneously, all the field components become zero indicating that a wave with components entirely transverse to the direction of propagation i.e. transverse electromagnetic wave, or TEM wave cannot exist in between the plates.

Two: With $E_z \neq 0$, $H_z = 0$, all the components are not zero indicating the possibility of the existence of a wave with its magnetic vector entirely normal to the propagation direction. This type of wave is called *E*-wave or transverse magnetic or *TM* wave.

Three: With $E_z = 0$, $H_z \neq 0$, there exist non-zero field components indicating possibility of a wave with its electric vector entirely normal to the propagation direction. This type of wave is called *H*-wave or transverse electric or TE wave.

Four: With $E_z \neq 0$, $H_z \neq 0$,the wave can exist in the guide, as all the field components are not zero. This type of wave is called *hybrid* or *mixed* wave.

The next step of the solution procedure involves, finding the fields, E_z and H_z , by solving the wave equations. They are partial differential equations, and their solution requires boundary conditions. As the tangential components of the electric field at the surface of a perfect conductor are zeros, and as the inner faces of the plates, which are assumed to be perfect conductors, are located at $x = 0$ and at $x = a$, the fields there must be zero i.e. $E_y = 0 = E_z$ at $x = 0$ and at $x = a$. These are the required boundary conditions and are highly useful in obtaining the expressions for the field components by solving the wave equations.

12.1.2. TM waves

The *TM* waves, as already mentioned, are characterized by the absence of magnetic field and the presence of electric field along the direction of wave propagation. It implies that the **H** vector is entirely transverse (synonym for perpendicular) to the direction of the wave travel i.e., thus, H_z =0 for a wave traveling in *z*-direction. When it comes to **E** vector, it exists along as well as normal to the direction of wave propagation. Now, to obtain an expression for E_z, the wave equation is solved, which is given by

$$\nabla^2 E_z = \mu\varepsilon \frac{\partial^2 E_z}{\partial t^2} \qquad (12.7)$$

If the time-variations and *z*-variations, both, are exponential, then the expression for E_z becomes

$$E_z\left(x,y,z,t\right) = E_z\left(x,y\right)e^{-\bar{\gamma}z}\, e^{j\omega t} \qquad (12.8)$$

After substitution of Eq. (12.8) in Eq. (12.7), and then some manipulation leads to

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \bar{\gamma}^2 E_z = -\omega^2 \mu\varepsilon\, E_z \qquad (12.9)$$

The wave travel is entirely confined to *z*-direction. As a consequence, the fields cannot have any variations along the *y*-direction, and hence, $\partial/\partial y = 0$. Incorporating this into Eq. (12.9), one can obtain that,

$$\frac{\partial^2 E_z}{\partial x^2} + \left(\bar{\gamma}^2 + \omega^2\mu\varepsilon\right)E_z = 0 \qquad (12.10)$$

Using $h^2 = \bar{\gamma}^2 + \omega^2\mu\varepsilon$, Eq.(12.10) can be recast as,

$$\frac{\partial^2 E_z}{\partial x^2} + h^2 E_z = 0 \qquad (12.11)$$

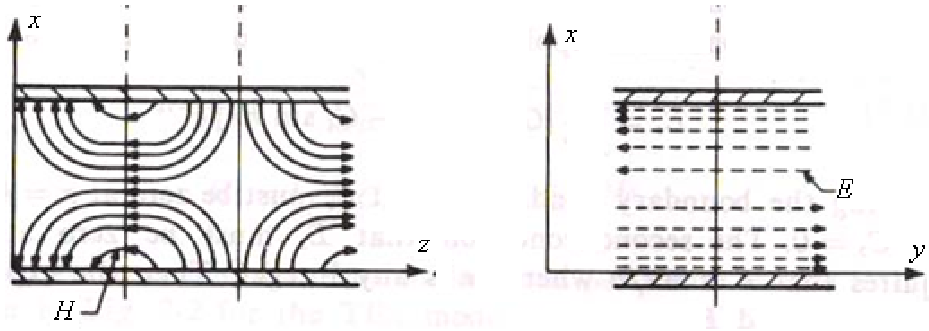


Figure 12.3 Fields between parallel planes for TM_{10} mode

The general solution to the above second order partial differential equation is,

$$E_z = \left(A\cos hx + B\sin hx\right)e^{-\bar{\gamma} z} \qquad (12.12)$$

where A and B are arbitrary constants whose values can be fixed using the boundary conditions, , $E_z = 0$ at $x = 0$ and at $x = a$, it can be obtained that

$$A = 0 \text{ and } h = m\pi/a, \quad m = 0,1,2\ldots \qquad (12.13)$$

After substituting the obtained values for arbitrary constants, the expression for the field becomes,

$$E_z = B\sin\left(\frac{m\pi}{a}x\right)e^{-\bar{\gamma} z} \qquad (12.14)$$

Here, B is an arbitrary constant and for mathematical convenience, its value is chosen as,

$$B = \frac{jm\pi}{\omega\varepsilon a}C_o$$

The C_o is another arbitrary constant. With the availability of expression for E_z , the other field components can be computed using Eqs.(12.2). After including the exponential time variations and z-variations, the complete set of the field components between the parallel plates, for *TM* wave, become

$$H_z = 0 \quad ; \quad E_z = C_o\frac{jm\pi}{\omega\varepsilon a}\sin\left(\frac{m\pi}{a}x\right)e^{-j\bar{\beta}z}e^{j\omega t} \qquad (12.15a)$$

$$E_x = C_o\frac{\bar{\beta}}{\omega\varepsilon}\cos\left(\frac{m\pi}{a}x\right)e^{-j\bar{\beta}z}e^{j\omega t} \quad ; \quad H_y = C_o\cos\left(\frac{m\pi}{a}x\right)e^{-j\bar{\beta}z}e^{j\omega t} \qquad (12.15b)$$

The expressions for fields of *TM* wave, available above, are illustrated in Figure 12.3, using flux lines for $m = 1$ i.e., TM_{10} mode.

12.1.3 TE waves

The TE waves are characterized by the absence of electric field and the presence of magnetic field along the direction of propagation of the wave. It implies that the **E** vector is entirely transverse to the direction of the wave travel. When it comes to **H** vector, it exists along as well as normal to the direction of wave propagation. In the present case, $E_z = 0$ and as the boundary conditions are not available on H_z, the component E_y, for which boundary conditions are available, is considered for obtaining the solution. The wave equation for E_y is given by,

$$\nabla^2 E_y = \mu\varepsilon \frac{\partial^2 E_y}{\partial t^2} \tag{12.16}$$

By following a procedure similar to that used in the previous section, one can arrive at

$$\frac{\partial^2 E_y}{\partial x^2} + h^2 E_y = 0 \tag{12.17}$$

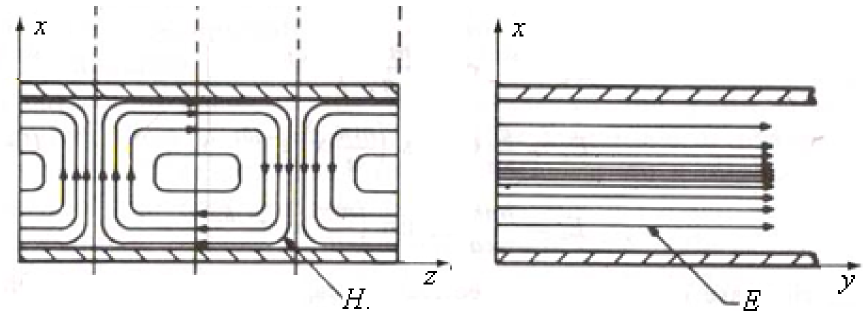


Figure 12.4 Fields between parallel planes for TE_{10} mode

The general solution of this equation is same as that for wave equation solved in the previous section. Using boundary conditions, $E_y = 0$ at $x = 0$ and $x = a$, one can get the particular or specific solution from the general one. The resultant specific solution then is

$$E_y = C_1 \sin\left(\frac{m\pi}{a}\right) x e^{-\bar{\gamma} z} \quad m = 1, 2, 3... \tag{12.18}$$

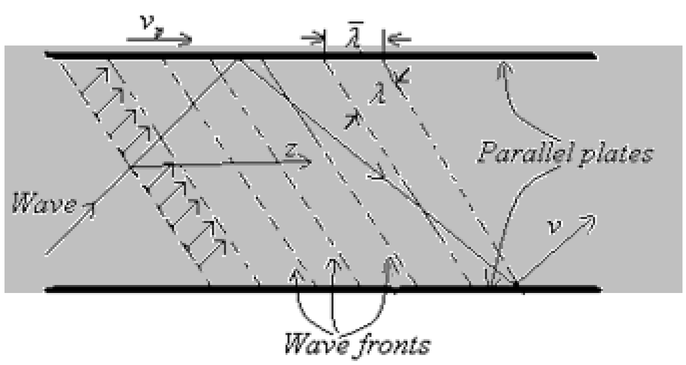


Figure 12.5 Illustrating wave propagation between parallel plates.

With the availability of expression for E_y, using second one of Eqs.(12.2b), H_z computed. From, E_z and H_z other components can be computed using the relations available in Eqs. (12.2), connecting transverse components to longitudinal ones. After including the exponential time variations and z-variations, the complete set of the field components for *TE* wave between the parallel plates become

$$E_z = 0 \quad ; \quad H_z = C_1 \frac{jm\pi}{\omega\mu a} \cos\left(\frac{m\pi}{a} x\right) e^{-j\bar{\beta} z} e^{j\omega t} \tag{12.19a}$$

$$E_y = C_1 \sin\left(\frac{m\pi}{a} x\right) e^{-j\bar{\beta} z} e^{j\omega t} \quad ; \quad H_x = -C_1 \frac{\bar{\beta}}{\omega\mu} \sin\left(\frac{m\pi}{a} x\right) e^{-j\bar{\beta} z} e^{j\omega t} \tag{12.19b}$$

The flux line representation of fields of *TE* wave are shown in Figure 12.4, for $m = 1$ i.e. TE_{10} mode. Until now, several aspects of propagation between plates are examined, and its behavior in between the parallel plates is illustrated in Figure 12.5. Note that the propagation is not through the metal of conductor plates: the wave actually passes through the hollow region in between the plates. The role of plates is merely to confine the wave to hollow region.

FILTER CHARACTERISTICS

Infinite parallel plate arrangement and waveguide systems, both, behave like high-pass filters. They admit and allow the wave to propagate through them only if the frequency of the wave is more than certain value known as cut-off frequency. Its value depends upon plate spacing/ guide dimensions, mode of wave and also the properties of the medium between plates/guide hollow region. Now, the terms, the cut-off frequency and its corresponding wavelength, called cut-off wavelength, phase shift constant and guide wavelength etc, which essentially describe the filtering nature of the guide system and waveguides are defined and explained in detail.

Cut-off frequency, f_c: It is the frequency above which frequency of the wave should be in order to get entry into the parallel plate system/waveguides for propagating further through them. Its value depends upon plates spacing/guide dimensions and on the wave mode.

Parallel plate guide: The cutoff frequency for parallel plate guide can be found as,

$$f_c = \frac{m}{2a\sqrt{\mu\varepsilon}} \qquad (12.38)$$

Cut-off wavelength, λ_c: Wavelength corresponding to cut-off frequency is called cut-off wavelength. It can be defined as wavelength below which wavelength of the wave should be in order to exist with in parallel plate system/waveguides for propagating further through them.

Its value, like the cutoff frequency, is related to plate spacing/ guide dimensions and mode numbers of the wave through it. However, unlike the cutoff frequency, it is independent of the constituent constants of the medium between the plates/hollow region.

Parallel plate guide: The cutoff wavelength for parallel plate guide can be found as,

$$\lambda_c = \frac{2a}{m} \qquad (12.41)$$

Phase-shift constant, $\bar{\beta}$***:*** The phase-shift constant, which indicates the phase change per unit distance, in the wave along the direction of propagation, is given by

$$\bar{\beta} = \omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \omega\sqrt{\mu\varepsilon}\sqrt{1-\frac{1}{\mu_r\varepsilon_r}\left(\frac{\lambda}{\lambda_c}\right)^2} \qquad (12.45)$$

In general, travelling waves are associated with change in phase and they undergo a phase change of 2π rad by the time they travel a distance equal to the wavelength, λ m. Therefore, phase change per unit distance or meter, which is phase shift constant, β becomes $2\pi/\lambda$.

Guide wavelength, $\bar{\lambda}$: The guide wavelength, indicated by $\bar{\lambda}$ or λ_g is wavelength along propagation direction and is by definition the distance between two consecutive points which differ in phase by 2π radians measured along the direction of wave propagation.

Both for parallel plate guide system and waveguides, it is given by,

$$\bar{\lambda} = \frac{2\pi}{\bar{\beta}} = \frac{\lambda}{\sqrt{\mu_r\varepsilon_r}\sqrt{1-(f_c/f)^2}} = \frac{\lambda}{\sqrt{\mu_r\varepsilon_r-(\lambda/\lambda_c)^2}} \qquad (12.46)$$

Here, it is assumed that region between plates/hollow region of guide is filled with a dielectric, having constitutive constants, ε_r and μ_r. From the expressions, Eq.(12.46), it can be observed that the guide wavelength varies with frequency. This dependence can be described as:

- Above cutoff, with increase in frequency of the wave, guide wavelength decreases and with decrease in frequency, guide wavelength increases. Just near cut-off frequencies, it assumes highest possible values.
- At cutoff, the guide wavelength is infinity and below cutoff, it assumes complex values indicating non-existence of traveling wave in the guide.

When region between parallel plates/guide's hollow region is free space, then the guide wavelength becomes,

$$\lambda_g = \frac{\lambda}{\sqrt{1-(\lambda/\lambda_c)^2}} = \frac{\lambda}{\sqrt{1-(f_c/f)^2}} \qquad (12.47)$$

It can be noticed that the first one is entirely in terms of wavelengths and the second one involves both frequencies as well as wavelength. These two are considered as standard expression for guide wavelength in an air-filled waveguides. The guide wavelength can also be framed in a more popular form, connecting all the three wavelengths as shown below:

$$\frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2} = \frac{1}{\lambda^2} \qquad (12.48)$$

Proof : Now, the derivation of relations for cut-off frequency and cut-off wavelength is undertaken. It is already known that the propagation constant $\bar{\gamma}$ can be related to the frequency, ω, of the wave, according to Eq. (12.3), as $\bar{\gamma}^2 = h^2 - \omega^2\mu\varepsilon$ or

$$\bar{\gamma} = \sqrt{h^2 - \omega^2 \mu\varepsilon} \qquad (12.49)$$

From the above relation, Eq.(12.49), it can be noticed that, depending upon the relative values of h^2 and $\omega^2\mu\varepsilon$, the propagation constant $\bar{\gamma}$ can be pure real or imaginary. When $h^2 > \omega^2\mu\varepsilon$ he propagation constant is pure real quantity, indicating that the waveguide is acting as a pure attenuator, without allowing wave motion, refusing entry to the wave. However, when $h^2 < \omega^2\mu\varepsilon$, the propagation constant is pure imaginary quantity, indicating that the waveguide is acting as a pure transmission line, without any attenuation to the wave. This must be the case to which the guiding system under consideration belongs, since it is given that the wave is already inside the guide and loss-less conditions are assumed prevailing, i.e., plates are assumed to be perfect conductors and region in between them is considered as perfect dielectric.

The change over in the behavior of the guiding system from pure attenuator to pure transmission line takes place as the frequency is increased from lower to higher values, when $h^2=\omega^2\mu\varepsilon$. The frequency of the wave corresponding to this relation is called cut-off frequency, ω_c=($2\pi f_c$). Hence, at cut-off frequency,

$$\omega_c^2 \mu\varepsilon = h^2 \qquad (12.50)$$

Combining h available in Eq.(12.13) with Eq.(12.50) results cut-off frequency for parallel plate guiding system, given in Eq.(12.38). Using h value available in Eq.(12.24) with Eq.(12.50) results cut-off frequency for rectangular waveguide, given in Eq.(12.39). And substituting h value available in Eqs.(12.32)and (12.37) with Eq.(12.50) results cut-off frequency for circular waveguides, given in Eqs.(12.40).

The corresponding wavelengths, cut-off wave lengths can be found using $\lambda_c = v/f_c$. Note that the expressions for cut-off frequency f_c and cut-off wavelength λ_c are same for both the types of waves i.e. *TE* and *TM* waves, in parallel plate guide and rectangular guides. However, for circular guides, *TE* and *TM* waves have different values. Another point worth mentioning here is, that the product of cut-off frequency and cut-off wavelength is equal to v ($=f_c \lambda_c = 1/\sqrt{\mu\varepsilon}$) but not to $c(= f\lambda = 1/\sqrt{\mu_o\varepsilon_o})$.

Next, the derivations pertaining to phase shift constant and guide wavelength are considered. In the present case, it is given that the wave is under loss-less conditions and it is already inside, so the propagation constant $\bar{\gamma}$ must be pure imaginary i.e.

$$\bar{\gamma} = \sqrt{h^2 - \omega^2 \mu\varepsilon} = j\bar{\beta} \qquad (12.51)$$

The phase shift constant, therefore, becomes

$$\bar{\beta} = \sqrt{\omega^2 \mu\varepsilon - h^2} \qquad (12.52)$$

Using h values available in Eqs. (12.13), (12.24), (12.32)and (12.37) the phase shift constants for parallel plate guide, rectangular guide and circular guides can be found from Eq.(12.52). From the basic definition of wavelength and phase shift constants, wavelength $\bar{\lambda}$ can be expressed as $\bar{\lambda} = 2\pi/\bar{\beta}$**.** Substituting the available expressions for $\bar{\beta}$ in this basic relation, the expressions for guide wavelength can be easily found.

Example 12.3 : Two parallel plane infinite conducting plates are separated by 2cm. Find the cut-off frequencies for m =1 and 2 when values of permeability and permittivity are (a) $\mu=\mu_o$, $\varepsilon=\varepsilon_o$ and(b) $\mu=\mu_o$, $\varepsilon=4\varepsilon_o$.

Solution:

(a) Permeability and permittivity are, $\mu=\mu_o$, $\varepsilon=\varepsilon_o$.

The cut-off frequency is,

$$f_c = \frac{m}{2a\sqrt{\mu\varepsilon}} = \frac{m \times 3\times10^{10}}{2a\sqrt{\mu_r\varepsilon_r}} = \frac{3\times10^{10}}{2\times2} = 0.75\times10^{10}\,\text{Hz}$$

This value is for m=1 and similarly, for m =2, it can be found that f_c = 1.5×10^{10}Hz

(b) Permeability and permittivity are, $\mu=\mu_o$, $\varepsilon=4\varepsilon_o$.

The cut-off frequency is,

$$f_c = \frac{m \times 3\times10^{10}}{2a\sqrt{\mu_r\varepsilon_r}} = \frac{3\times10^{10}}{2\times2\times2} = 0.375\times10^{10}\,\text{Hz} \text{ for } m=1$$

This value is for m=1 and similarly, for m =2, it can be found that f_c = 0.75×10^{10}Hz

Example 12.4: Two parallel plane infinite conducting plates are separated by 4cm. Find the cut-off wavelengths for m =1 and 2 when values of permeability and permittivity (a) $\mu=\mu_o$, $\varepsilon=\varepsilon_o$ and(b) $\mu=\mu_o$, $\varepsilon=4\varepsilon_o$.

Solution:

(a) Permeability and permittivity are $\mu=\mu_o$, $\varepsilon=\varepsilon_o$

The cut-off wavelength, is

$$\lambda_c = \frac{2a}{m} = \frac{2\times4}{m} = \frac{2\times4}{1} = 8\text{cm}$$

This value is for m=1 and similarly, for m =2, it can be found that λ_c = 4cm. As the cut-off wavelength is independent of medium properties, previous results are valid even for (b) $\mu=\mu_o$, $\varepsilon=4\varepsilon_o$.

Example 12.5: Two parallel plane infinite conducting plates are separated by 4cm. Assuming frequency of the wave as 9.0GHz, find the wavelength along the propagation direction, for m =1 and 2 when values of permeability and permittivity are (a) $\mu=\mu_o$, $\varepsilon=\varepsilon_o$ and(b) $\mu=\mu_o$, $\varepsilon=4\varepsilon_o$..

Solution:

(a) For the given values of $\mu=\mu_o$, $\varepsilon=\varepsilon_o$ and f=9.0GHz, it can be found that,

$$\omega^2\mu\varepsilon = \frac{4\pi^2 f^2 \mu_r \varepsilon_r}{c^2} = \frac{4\pi^2 \times 9^2 \times 10^{18}}{3^2 \times 10^{20}} = (0.6\pi)^2$$

The wavelength along the propagation direction then becomes,

$$\bar{\lambda} = \frac{2\pi}{\sqrt{\omega^2\mu\varepsilon - (m\pi/a)^2}} = \frac{2\pi}{\sqrt{(0.6\pi)^2 - (\pi/4)^2}} = 2.141\text{cm}$$

This value is for m=1 and similarly, for m =2, it can be found that $\bar{\lambda}$ = 6.03cm.

(b) For the given values of $\mu=\mu_o$, $\varepsilon=4\varepsilon_o$ and f=9.0GHz, it can be found that,

$$\omega^2\mu\varepsilon = \frac{4\pi^2 f^2 \mu_r \varepsilon_r}{c^2} = \frac{4\pi^2 \times 9^2 \times 10^{18} \times 4}{3^2 \times 10^{20}} = (1.2\pi)^2$$

The wavelength along the propagation direction then becomes,

$$\bar{\lambda} = \frac{2\pi}{\sqrt{\omega^2\mu\varepsilon - (m\pi/a)^2}} = \frac{2\pi}{\sqrt{(1.2\pi)^2 - (\pi/4)^2}} = 2.466\text{cm}$$

This value is for m=1 and similarly, for m =2, it can be found that $\bar{\lambda}$ = 2.91cm.

12.4 MODAL PROPAGATION

The electromagnetic energy propagation in between parallel plates, and along the waveguide is in the form of certain definite field patterns, known as 'modes'. This is an important and special feature of the energy propagation in guided waves and waveguides. These field patterns or modes are described by mentioning the transverse nature of wave i.e. *TE* or *TM* along with two subscript numbers, m and n.

Parallel plate guide: The guided wave propagation is in the form of modes, as is evident from Figures 12.2 and 12.3. These modes are distinguished with type of mode, that is *TE* or *TM*, and with mode numbers as subscripts, denoted with m and 0 in that *order*. Transverse electric modes are denoted as TE_{m0} whereas transverse magnetic modes by TM_{m0}. Certain points worth mentioning regarding modal propagation are given below:

- Subscripts: For *TE* wave, m can assume any integer value, from 1 onwards. For TM wave, however, m can assume any integer value from 0 onwards. Actually, the subscripts for modes are m and n and they are supposed to indicate the no. of half wavelengths along transverse directions normal to direction of propagation. In the present case, as there is no wave motion in one of the transverse directions, y-direction, the second subscript has become zero.

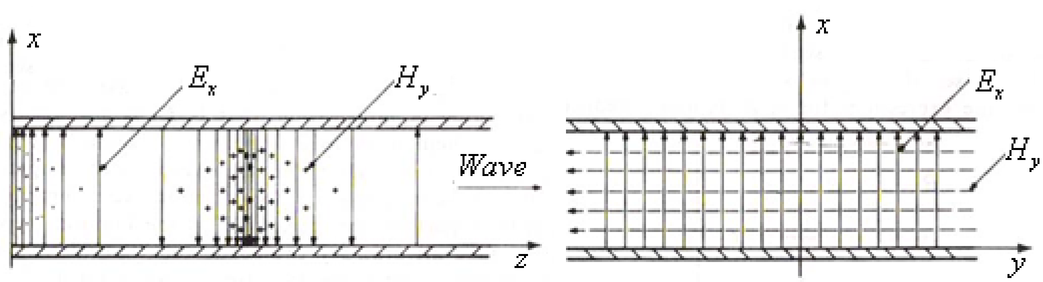


Figure 12.10 Fields between parallel planes for *TEM* or TM_{00} mode

- Principal mode: A wave with, both E_z and H_z equal to zero, is called principal wave. In this, electric and magnetic vectors, both, are normal to direction of propagation, and hence, it is a TEM wave. This type of wave comes into being, when m = 0 for TM mode and hence, its mode is TM_{00}. Substituting m = 0 in Eq.(12.19), the field components for the principal wave can be obtained as,

$$E_z = 0 \qquad ; \qquad H_z = 0 \qquad (12.53a)$$

$$E_x = C_o \frac{\bar{\beta}}{\omega\varepsilon} e^{-j\bar{\beta}z} e^{j\omega t} \qquad ; \qquad H_y = C_o e^{-j\bar{\beta}z} e^{j\omega t} \qquad (12.53b)$$

The fields of principal wave, TM_{00} are shown in Figure 12.10, using flux-line representation. The concept and utility of flux lines or field lines in the representation of the fields are already introduced and explained. As the wave carries both electric and magnetic fields, its representation in guide involves both types of flux lines, i.e. electric as well as magnetic lines. From the basic nature of fields, it can be noticed that electric and magnetic lines never cross each other. The magnetic lines are always closed curves. Both the categories of the lines can be sketched from the expressions of the fields in those regions.

Apart from the general usefulness i.e. graphical representation of fields, the flux line representation of fields in waveguides has another important application. It is useful as well as used in placing the probe or loop at appropriate place to excite the required mode in the guide.

12.5 DISPERSION CHARACTERISTICS

Electrical transmission media, in which the velocity of the wave depends upon its frequency, are called dispersive media. Parallel plates, hollow-pipe waveguides, which are now under consideration and optic fiber cables, used to transmit signals at light frequencies, are examples for this type. If the velocity is frequency independent, then such media are termed as non-dispersive media. The parallel wire transmission lines, coaxial cables, and also free space fall in this category.

As parallel plates and hollow-pipe waveguides are dispersive, the waves that are carried by them, TE and TM, are referred as *dispersive* waves. Similarly, waves carried by non-dispersive media, TEM waves, are called *non-dispersive* waves.

To describe the wave phenomenon, parameters like wavelength, time period, frequency etc. are required. Along with them, the phase and group velocities also have some importance in wave theory, particularly, with respect to waves in non-TEM mode. Here, both these velocities are considered with respect to parallel plates and waveguides.

12.5.1 *Phase velocity*:

The phase velocity, v_p is defined as the velocity with which the equi-phase surfaces propagate along the guiding system. It can be found using,

$$v_p = \frac{\omega}{\bar{\beta}} \qquad (12.55)$$

Parallel plate waveguides: Now, consider the phase velocity of wave travelling in between parallel plates/ in waveguides. An expression for phase shift constant of this wave has already been derived, and available in Eq. (12.44). Substituting this expression into the Eq. (12.55), one can get

$$v_p = v\left[1 - \frac{1}{\mu_r\varepsilon_r}\left(\frac{\lambda}{\lambda_c}\right)^2\right]^{-1/2} = v\left[1 - \left(\frac{f_c}{f}\right)^2\right]^{-1/2} \qquad (12.56)$$

In the case of air-filled region between the plates, then ε_r=1 and μ_r =1, and the expression for phase velocity becomes

$$v_p = c\left[1-\left(\frac{\lambda}{\lambda_c}\right)^2\right]^{-1/2} = c\left[1-\left(\frac{f_c}{f}\right)^2\right]^{-1/2} \quad (12.57)$$

Table 12.1 Properties of guided wave propagation and waveguides.

S.No	Parameter	$f = f_c$	$f \to \infty$
1.	Phase velocity, $\bar{v}$	∞	$\bar{v} = v = 1/\sqrt{\mu\varepsilon}$
2.	Group velocity, v_g	0	v
3.	Guide wavelength, $\bar{\lambda}$	∞	$\bar{\lambda} = \lambda = 2\pi/\omega\sqrt{\mu\varepsilon}$
4.	Phase shift constant, $\bar{\beta}$	0	$\bar{\beta} = \omega\sqrt{\mu\varepsilon}$

12.5.2 *Group velocity* :

The group velocity, v_g is the velocity with which the signal or group of frequencies, denoting intelligence, travels through the system. It is given by

$$v_g = \frac{d\omega}{d\bar{\beta}} \quad (12.58)$$

Parallel plates and waveguides: Now, consider the group velocity of wave travelling in between parallel plates/ in waveguides. By differentiating the Eq. (12.24), which relates the frequency with phase shift constant, one can obtain the relation for group velocity as follows:

$$\frac{d\bar{\beta}}{d\omega} = \frac{1}{2}\left[\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon\right]^{-1/2} 2\omega\mu\varepsilon = \omega\mu\varepsilon\left[\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon\right]^{-1/2} \quad (12.59)$$

Inverting the relation in Eq. (12.59), the group velocity of the wave between parallel plates can be obtained as,

$$v_g = \frac{d\omega}{d\bar{\beta}} = \frac{\sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}}{\omega\mu\varepsilon} = \frac{v^2\sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}}{\omega} \quad (12.60)$$

The relation in Eq.(12.60) can also be put in terms of cut-off values by simple manipulation as,

$$v_g = v\left[1-\frac{1}{\mu_r\varepsilon_r}\left(\frac{\lambda}{\lambda_c}\right)^2\right]^{1/2} = v\left[1-\left(\frac{f_c}{f}\right)^2\right]^{1/2} \quad (12.61)$$

In case, if the region in between the plates is free space, then

$$v_g = c\left[1-\left(\frac{\lambda}{\lambda_c}\right)^2\right]^{1/2} = c\left[1-\left(\frac{f_c}{f}\right)^2\right]^{1/2} \quad (12.62)$$

From the expressions of phase and group velocities, one can easily notice that their product is equal to square of velocity of unbounded wave, i.e. $\bar{v}v_g = v^2$. In case hollow region is free space, this product becomes c^2. Various parameters of wave propagation in between the plates are given in the Table 12.1. In the above expressions, Eqs.(12.61) and (12.62), the parameter v is given by $v = 1/\sqrt{\mu\varepsilon}$ and it can be considered as the velocity of the wave in unbounded medium having constants, μ and ε. It is related to free space velocity, c through $v = c/\sqrt{\mu_r \varepsilon_r}$.

Note that, in the case of air-filled waveguide, as the frequency is increased from the cut-off to infinity, guide wavelength and phase velocity vary from infinity to their free space value where as the group velocity varies from zero to its free space value. Also note that the phase velocity and group velocity are same for TEM wave. None of these two depend upon the frequency and so TEM wave is non-dispersive.

As just seen, in parallel plate guiding system and in waveguides, as they are dispersive, the wavelength along the length of guide, velocities of the wave, both phase as well as the group, varies with the frequency. This dispersion phenomenon is mostly unwanted because, in the signal, it spoils the original phase relation between different frequency components, as it travels down the guide. Ultimately, it leads to distortion and complete or partial loss of information. Here, various parameters, and also expressions for them, pertaining to the dispersive nature of waveguides are introduced and described.

Example 12.10: Two parallel plane infinite conducting plates are separated by 4cm. Find the wave velocity and group velocity of 9.0GHz wave for m =1 and 2 when values of permeability and permittivity are (a) $\mu=\mu_o$, $\varepsilon=\varepsilon_o$ and(b) $\mu=\mu_o$, $\varepsilon=4\varepsilon_o$.

Solution:

(a) For the given values of $\mu=\mu_o$, $\varepsilon=\varepsilon_o$ and f=9.0GHz, it can be found that,

$$\omega^2\mu\varepsilon = \frac{4\pi^2 f^2 \mu_r\varepsilon_r}{c^2} = \frac{4\pi^2\times 9^2\times 10^{18}}{3^2\times 10^{20}} = (0.6\pi)^2$$

The wave velocity, can be found as,

$$\bar{v} = \frac{\omega}{\sqrt{\omega^2\mu\varepsilon-(m\pi/a)^2}} = \frac{2\pi\times 9\times 10^9}{\sqrt{(0.6\pi)^2-(\pi/4)^2}} = 3.30\times 10^{10}\,\text{cm/s}$$

This value is for m=1 and similarly, for m =2, it can be found that $\bar{v}$ = 5.4×10^{10}cm/s.

As the wave is in free space, $v=c$ and the group velocity can be found as,

$$v_g = \frac{v^2}{\bar{v}} = \frac{c^2}{\bar{v}} = \frac{(3\times 10^{10})^2}{3.30\times 10^{10}} = 2.72\times 10^{10}\,\text{cm/s}$$

This value is for m=1 and similarly, for m =2, it can be found that v_g = 1.6×10^{10}cm/s.

(b) For the given values of $\mu=\mu_o$, $\varepsilon=4\varepsilon_o$ and f=9.0GHz, it can be found that,

$$\omega^2\mu\varepsilon = \frac{4\pi^2 f^2 \mu_r\varepsilon_r}{c^2} = \frac{4\pi^2\times 9^2\times 10^{18}\times 4}{3^2\times 10^{20}} = (1.2\pi)^2$$

The wave velocity, can be found as,

$$\bar{v} = \frac{2\pi\times 9\times 10^9}{\sqrt{(1.2\pi)^2-(\pi/4)^2}} = 2.20\times 10^{10}\,\text{cm/s}$$

This value is for m=1 and similarly, for m =2, it can be found that $\bar{v}$ = 2.6×10^{10}cm/s.

As the wave is in dielectric medium with ε_r=4, velocity, $v=c/2$ and the group velocity, can be found as,

$$v_g = \frac{v^2}{\bar{v}} = \frac{(1.5\times 10^{10})^2}{2.20\times 10^{10}} = 1.02\times 10^{10}\,\text{cm/s}$$

This value is for m=1 and similarly, for m =2, it can be found that v_g = 0.86×10^{10}cm/s.

12.6. IMPEDANCES OF WAVEGUIDES

It was Oliver Heaviside, who coined the term *impedance* for the first time in nineteenth century, to describe the complex ratio **V**/**I** in AC circuits. Later, Schelkunoff extended this concept to electromagnetic fields, in a systematic way to describe the ratio of electric to magnetic fields. In his opinion, impedance should be considered as a combined characteristic of field and medium.

In waveguide theory, two types of impedances are encountered: wave impedance and characteristic impedance. The first one is borrowed from wave theory and it is related to ratio of electric field to magnetic field of a traveling wave, where as the second impedance is related to the description of power flow along the length of the guide. Both these impedances are described in detail and corresponding mathematical expressions are derived hereunder.

12.6.1. Wave Impedance

The wave impedance is usually denoted by $\mathbf{Z}_z$ and for waveguides, it is defined as the ratio of total transverse electric field strength to total transverse magnetic field strength. Mathematically,

$$\mathbf{Z}_z = \frac{\text{Total transverse electric field}}{\text{Total transverse magentic field}} = \frac{\mathbf{E}_{trans}}{\mathbf{H}_{trans}} \qquad (12.63)$$

In case of rectangular waveguides, when they are lying along the *z*-axis, the *x*- and *y*-components of the fields constitute the transverse components as they are normal to the propagation direction. The total transverse electric field then is $\sqrt{(\mathbf{E}_x^2 + \mathbf{E}_y^2)}$ whereas the total transverse magnetic field is $\sqrt{(\mathbf{H}_x^2 + \mathbf{H}_y^2)}$. Thus, when the guide is lying along the *z*-axis, wave impedance, according to definition, becomes

$$\mathbf{Z}_z = \frac{\mathbf{E}_{trans}}{\mathbf{H}_{trans}} = \frac{\sqrt{\mathbf{E}_x^2 + \mathbf{E}_y^2}}{\sqrt{\mathbf{H}_x^2 + \mathbf{H}_y^2}} \qquad (12.64)$$

In case of circular waveguides, lying along the *z*-axis, the ρ- and ϕ-components of fields are normal to propagation direction, and hence, they are the transverse components. Thus, when the guide is lying along *z*-axis, its wave impedance becomes,

$$\mathbf{Z}_z = \frac{\mathbf{E}_{trans}}{\mathbf{H}_{trans}} = \frac{\sqrt{\mathbf{E}_\rho^2 + \mathbf{E}_\phi^2}}{\sqrt{\mathbf{H}_\rho^2 + \mathbf{H}_\phi^2}} \qquad (12.65)$$

It can be shown that the value of the impedance depends upon the mode of the wave traveling in the guide and it is given, for both the types of waveguides, by

$$\mathbf{Z}_z = \frac{\omega\mu}{\bar{\beta}} = \eta \Big/ \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = \eta \Big/ \sqrt{1 - \frac{1}{\mu_r \varepsilon_r}\left(\frac{\lambda}{\lambda_c}\right)^2} \quad \text{for TE mode}$$

$$= \frac{\bar{\beta}}{\omega\varepsilon} = \eta\sqrt{1 - \left(\frac{f_c}{f}\right)^2} = \eta\sqrt{1 - \frac{1}{\mu_r \varepsilon_r}\left(\frac{\lambda}{\lambda_c}\right)^2} \quad \text{for TM mode} \qquad (12.66)$$

Here η is intrinsic impedance and it is related to the constants, μ and ε , of the of the hollow region of the guide through $\eta=\sqrt{(\mu/\varepsilon)}$ Ω. Note that, for free space or air, the intrinsic impedance is 120π Ω i.e., η_o = 120π Ω.

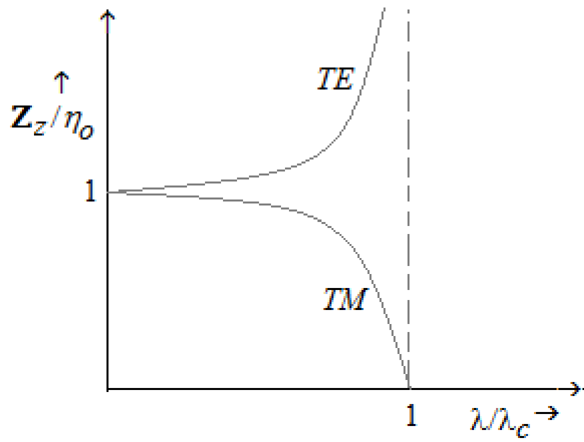


Figure 12.13 Illustrating the variation of wave impedance with wavelength.

In general, wave impedance is a characteristic type of wave, TEM, TE, and TM and, it usually depends upon the type of line or guide, the material and the operating frequency. This parameter of the waveguide medium can be likened, conceptually, to the intrinsic impedance of the free-space medium.

12.6.2. Characteristic impedance

The concept of characteristic impedance has been brought into and applied to waveguides from theory of transmission lines. It is defined in several equivalent ways, in transmission line theory, in terms of line voltages, currents and power through the line. Voltage-current formula, power-current formula and power-voltage formula respectively are

$$\mathbf{Z}_o(\mathbf{V},\mathbf{I}) = \frac{\mathbf{V}}{\mathbf{I}} \ , \ \mathbf{Z}_o(\mathbf{P},\mathbf{I}) = \frac{2\mathbf{P}}{\mathbf{I}\,\mathbf{I}^*} \ \ \& \ \ \mathbf{Z}_o(\mathbf{P},\mathbf{V}) = \frac{\mathbf{V}\,\mathbf{V}^*}{2\mathbf{P}} \qquad (12.67)$$

where **V** and **I** are voltage and current and **P** is the power flowing over the line, when extended to infinity, all are complex quantities representing peak phasors.

All the above definitions, give same value of characteristic impedance, when applied to a low frequency TEM wave carrying line. However, when applied to waveguides, different definitions give different values of characteristic impedance. It is mainly due to the lack of a unique definition for voltage and current in terms of fields in waveguide. Under such circumstances, it is quite possible, for the above mentioned different defining relations, which are in terms of voltages and currents, to give different values of characteristic impedance.

When the voltages and currents are computed in a commonly and widely followed method, one can obtain the following expressions for the characteristic impedance.

- In case of rectangular waveguides, for dominant mode:

  Voltage-current formula gives

$$\mathbf{Z}_o(\mathbf{V},\mathbf{I}) = \frac{\pi}{2}\frac{b}{a}\mathbf{Z}_z(TE)$$
$$= 60\pi^2\frac{b}{a}\sqrt{\frac{\mu_r}{\varepsilon_r}}\frac{\lambda_g}{\lambda} = 592\frac{b}{a}\sqrt{\frac{\mu_r}{\varepsilon_r}}\frac{\lambda_g}{\lambda} \qquad (12.68a)$$

  Power-current formula and power-voltage formula respectively give

$$\mathbf{Z}_o(\mathbf{P},\mathbf{I}) = \frac{\pi}{4}\mathbf{Z}_o(\mathbf{V},\mathbf{I}) \;\;\&\;\; \mathbf{Z}_o(\mathbf{P},\mathbf{V}) = \frac{4}{\pi}\mathbf{Z}_o(\mathbf{V},\mathbf{I}) \qquad \text{12.68b)}$$

• In case of circular waveguides, for dominant mode voltage-current formula, power-current formula and power-voltage formula respectively gives

Voltage-current formula gives,

$$\mathbf{Z}_o(\mathbf{V},\mathbf{I}) = 520\frac{\lambda_g}{\lambda},\;\; \mathbf{Z}_o(\mathbf{P},\mathbf{I}) = 354\frac{\lambda_g}{\lambda} \;\;\&\;\; \mathbf{Z}_o(\mathbf{P},\mathbf{V}) = 764\frac{\lambda_g}{\lambda} \qquad (12.69)$$

It can also be noted that, the characteristic impedance is mode dependent. The reason is not difficult to find: voltages and currents are mode dependent, and hence, TE and TM waves have different values.

The primary utility of characteristic impedance is, to set the value of load impedance for reflection-less transmission. When its value is not unique, naturally, doubt arises regarding the value which one has to select for matching purpose. The usual practice is, to match the wave-guide to uniquely defined impedance, to use the impedance value that gives best agreement between theory and experimental data.

12.7. ATTENUATION

In the analysis of wave in between parallel conductor planes or in waveguides, it was assumed loss-free conditions, mainly, to simplify the analysis procedure. The wave, while travelling through parallel conductor plane system or waveguide, comes in contact with the conductor planes as well as the hollow region between the conductor planes. The planes are assumed to be perfect conductors and the in between region is considered as a perfect dielectric without any conductivity. A perfect conductor and a perfect dielectric can neither absorb nor dissipate the power and as result loss free situation prevailed.

But in practice, conductivity of the planes is finite, not infinite and that of the in between region is not zero, a nonzero. As a consequence, the wave, while traveling through the parallel plane guiding system, gets absorbed, however small it may be, by the conductor planes as well as the in between region. It ultimately leads to attenuation of the wave, called *dissipative* attenuation.

The dissipative attenuation has two components: *dielectric losses* and *conductor losses*. Due to nonzero conductivity, the region in between the plates absorbs power from the wave leading to its attenuation, which is accounted for by dielectric loss. Similarly, due to finite conductivity, the plates absorb power from the wave, attenuating it, and it is accounted by conductor loss.

In case of non-TEM wave carrying systems, in addition to dissipative attenuation, another type, called *reflective* attenuation also exists. This comes into being when the wavelength of wave is not small enough to get admitted into guiding system. Magnitude wise, it is enormously large when compared to dissipative attenuation. All the types of attenuations are placed in the Table 12.4.

Table 12.4 Attenuation properties of waveguides.

S.No.	**Reflective attenuation,** dB/m	**Dissipative attenuation,** Np/m	
1.	54.6	Dielectric losses	Conductor losses

		$\bar{\alpha}_d = \dfrac{\beta^2 \tan\delta}{2\bar{\beta}}$	$\bar{\alpha}_c = \dfrac{\text{power dissipated/unit length}}{2\times\text{power flow down the guide}}$
2.	$f < f_c$	$f > f_c$	
3.	Huge	Very low	

Now, the analysis of the wave when it is traveling through a practical, not ideal loss-free guide, is undertaken.

12.7.1 Reflective Attenuation:

When wavelength is more than the cut-off wavelength i.e., $\lambda > \lambda_c$, the wave cannot enter into the guiding plates or waveguides. This behavior of the guiding system is taken into account by attributing large amount of reflective attenuation to the guiding system. The important features of this attenuation are:

- When a guiding system is excited with a wave whose wavelength is more than cut-off value, i.e., $\lambda > \lambda_c$, the electric and magnetic fields of the wave decay in the guide exponentially with distance at a very rapid rate due to huge amount of attenuation.
- The resultant attenuation depends only on the ratio λ/λ_c i.e. free space wavelength to the cut-off wavelength.
- The attenuation, however, is independent of properties of guiding plates or in-between region. This feature is very much unlike the dissipative attenuation, which is dependent upon the conductivity of the guiding walls and the hollow region in between the walls.
- The exact relation for attenuation per unit length in dB is

$$\alpha = \frac{54.6}{\lambda_c}\sqrt{1-\left(\frac{\lambda_c}{\lambda}\right)^2} \qquad (12.70)$$

When the wavelength is much greater than its cut-off value, the above formula can be approximated to

$$\alpha \approx \frac{54.6}{\lambda_c} \qquad (12.71)$$

- These relations apply to all modes of propagation and it can be observed that when λ/λ_c large, the attenuation is large and substantially independent of frequency.

12.7.2 Dissipative attenuation:

When wavelength is less than its cut-off value, i.e., $\lambda < \lambda_c$, the wave can exist inside and travel through the guiding system. While traveling, some of its energy gets absorbed by the walls due to their finite conductivity and also by the hollow region due to its non-zero conductivity, resulting in dissipative attenuation. Its two components, dielectric loss and conductor loss, and both are described and discussed about, one after another here.

Attenuation due to dielectric loss: The attenuation the wave undergoes while it is traveling through the guide, due to energy absorption by the lossy dielectric region of the guiding system is called *dielectric loss*. It can be shown that the dielectric loss is

$$\alpha_d = \frac{\beta^2 \tan\delta}{2\bar{\beta}} \text{ Np/m} \qquad (12.72)$$

Note that if the dielectric is a perfect one i.e. without conductivity, then this loss would not occur.

Proof: If the region in between the plates is a perfect dielectric i.e. with zero conductivity, then its permittivity, ε is, let us say $\varepsilon_o\varepsilon_r$ i.e., $\varepsilon = \varepsilon_o\varepsilon_r$, as shown in Figure 12.13(a). When the region is an imperfect dielectric i.e. with nonzero conductivity, let us say, σ, then its permittivity, ε becomes $\varepsilon_o\varepsilon_r(1 - j\tan\delta) = \varepsilon(1 - j\tan\delta)$, where tan δ is called loss tangent and is given by $\tan\delta = \sigma/\omega\varepsilon$, as shown in Figure 12.13(b). Note that loss tangent assumes zero value for perfect dielectric media.

In case of perfect dielectric in between the plates, and wave inside the guides, the propagation constant, from Eq. (12.22), is

$$\bar{\gamma} = j\bar{\beta} = \sqrt{h^2 - \omega^2\mu\varepsilon} \qquad (12.73)$$

However, with imperfect dielectric in between the plates, the propagation constant becomes

$$\bar{\gamma} = \sqrt{h^2 - \omega^2\mu\varepsilon\left(1 - j\tan\delta\right)} = \sqrt{h^2 - \omega^2\mu\varepsilon + j\omega^2\mu\varepsilon\tan\delta} \qquad (12.74)$$

In the above relation, note that the expression for permittivity used is that of an imperfect dielectric, given by $\varepsilon(1 - j\tan\delta)$. The last term in Eq. (12.74), in general, is very small, because of low conductivity of the dielectric region. Now if, $h^2 - \omega^2\mu\varepsilon = a^2$ and $\omega^2\mu\varepsilon\tan\delta = x^2$ then the propagation constant in Eq. (12.74) can be written as

$$\bar{\gamma} = \sqrt{a^2 + jx^2}$$

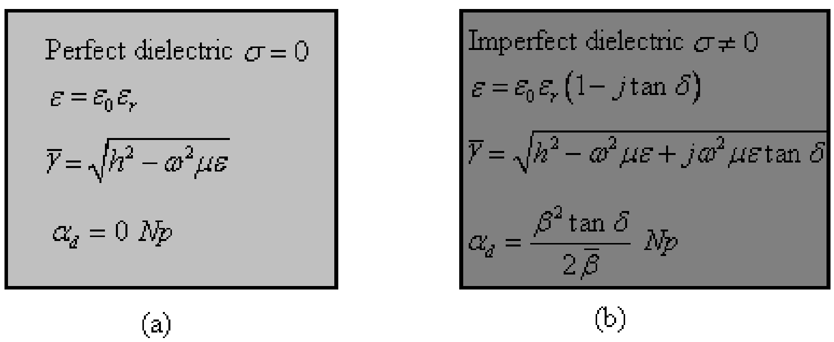


Figure 12.13 Dielectric loss in (a) perfect dielectric and in (b) imperfect dielectric.

In practice, as already mentioned x is very small and $x \ll a$. In such case, the propagation constant can be approximated, with the help of Taylor's series expansion, as

$$\bar{\gamma} \approx a + j\frac{1}{2}\frac{x^2}{a} = \sqrt{h^2 - \omega^2\mu\varepsilon} + j\frac{1}{2}\frac{\omega^2\mu\varepsilon\tan\delta}{\sqrt{h^2 - \omega^2\mu\varepsilon}} \qquad (12.75)$$

From Eq. (12.73), as $j\bar{\beta} = \sqrt{h^2 - \omega^2\mu\varepsilon}$ the above Eq. (12.75) can be expressed in terms of phase shift constant, $\bar{\beta}$ as

$$\bar{\gamma} = j\bar{\beta} + j\frac{1}{2}\frac{\omega^2\mu\varepsilon\tan\delta}{j\bar{\beta}} = \frac{\omega^2\mu\varepsilon\tan\delta}{2\bar{\beta}} + j\bar{\beta}$$

But, from definition, the real part of propagation constant is attenuation constant. Thus, the attenuation constant due to dielectric loss becomes

$$\alpha_d = \frac{\omega^2\mu\varepsilon\tan\delta}{2\bar{\beta}} = \frac{\beta^2\tan\delta}{2\bar{\beta}} \text{ Np/m.} \qquad (12.76)$$

In the above expression, β, phase shift constant for TEM wave, is substituted for $\omega\sqrt{\mu\varepsilon}$. This relation for attenuation is valid for all types of waves, i.e. TEM, TE and TM waves. However, in the case of TEM waves, $\bar{\beta}=\beta=\omega\sqrt{\mu\varepsilon}$, making the relation the attenuation constant as

$$\alpha_d = \frac{\omega\sqrt{\mu\varepsilon}}{2}\tan\delta \ \text{Np/m.}$$

It can be observed that the dielectric loss becomes nil when the conductivity or loss tangent of the dielectric is zero. The above relations for attenuation constant due to the dielectric loss are most general and can be used with any type of guiding system including waveguides.

Attenuation due to conductor loss: While traveling through the guide, wave also undergoes considerable amount of attenuation due to absorption by the conducting plates. The attenuation due to conductor loss can be found as

$$\alpha_c = \frac{\text{power dissipated/unit length}}{2\times\text{power flow down the guide}}\ \text{Np/m} \qquad (12.77)$$

Note that if the plates are perfect conductors, then this type of attenuation would not have occurred. Thus, it is due to *finite* conductivity of the walls of waveguide.

Proof: The derivation of the above formula is based on the principles of transmission line theory. Consider a finite length transmission line and let us suppose the voltage and current phasors along the line, when it is extended to infinity, are

$$V = V_0 e^{-\alpha z} e^{-j\beta z} \quad \text{and} \quad I = I_0 e^{-\alpha z} e^{-j\beta z}$$

Then the average power transmitted is

$$P_{av} = \frac{1}{2}\text{Re}\left\{VI^*\right\} = \frac{1}{2}\text{Re}\left\{V_0 I_0^*\right\} e^{-2\alpha z}$$

The rate of decrease of transmitted power along the line will be

$$-\frac{\partial P_{av}}{\partial z} = +2\alpha P_{av}$$

The decrease of transmitted power per unit length of line is

$$-\Delta P_{av} = 2\alpha P_{av}$$

And, this is the power dissipated per unit length. Solving the above equation for the attenuation constant

$$2\alpha = \frac{-\Delta P_{ave}}{P_{ave}} = \frac{\text{Power lost per unit length}}{\text{Power transmitted}}$$

It leads to the expression for attenuation constant

$$\alpha = \frac{\text{Power lost per unit length}}{2\times\text{Power transmitted}}$$

This relation is a most general one and it can be used to find the attenuation in parallel plate system as well as in waveguides.

12.7.3. Attenuation in Parallel plate guide

Attenuation for TEM wave : Now the Eq. (12.77) is applied to parallel plate guiding system to find conductor losses, first for TEM wave and then for TE/TM waves. The field components for the principal or TEM wave, from Eq.(12.51), are

$$H_z = 0 \quad ; \quad E_z = 0$$

$$E_x = \left(\bar{\beta}/\omega\varepsilon\right) C_o e^{-j\bar{\beta}z} e^{j\omega t}; \quad H_y = C_o e^{-j\bar{\beta}z} e^{j\omega t}$$

The surface current density over inner face of each plate can be computed from, **K**= $\hat{\boldsymbol{n}}$×**H** and its amplitude can be found as, K=C_o. The loss per m^2 on each plate is $K^2R_s/2=C_o^2R_s/2$. Here, R_s is the surface resistance of the conducting plane and it is given by

$$R_s = \sqrt{\frac{\omega\mu_m}{2\sigma_m}}\Omega$$

The total loss in both lower and upper plates per meter length with in a width of b meters becomes, $C_o^2R_sb$. Thus, the power loss per unit length becomes, $C_o^2R_s$. According to Poynting theorem, the power transmitted down the guide per unit cross sectional area is Re(**E**×**H***)/2. In the present case, the fields are at right angles and in time phase. Their ratio is η, intrinsic impedance of the in between region. Thus, the power flow per unit cross sectional area becomes $\eta C_o^2/2$. When the spacing is a and width is b , the cross sectional area becomes, ba . Thus, power transmitted through this area = $\eta C_o^2ba/2$. With availability of power loss and power flow, the attenuation factor can now be computed, using Eq.(12.77), as

$$\alpha = \frac{C_o^2R_sb}{2\times(1/2)\eta C_o^2ba} = \frac{1}{\eta a}\sqrt{\frac{\omega\mu_m}{2\sigma_m}} \qquad \text{Np/m} \qquad (12.78)$$

Attenuation for TE/TM wave: Let us consider first the TE wave and, its field components from Eq. (12.19) are:

$$E_z = 0 \qquad ; \qquad H_z = -\frac{\bar{\beta}}{\omega\mu}C_1\cos\left(\frac{m\pi}{a}x\right)e^{-j\bar{\beta}z}e^{j\omega t}$$

$$E_y = C_1\sin\left(\frac{m\pi}{a}x\right)e^{-j\bar{\beta}z}e^{j\omega t} \quad ; \quad H_x = \frac{jm\pi}{\omega\mu a}C_1\sin\left(\frac{m\pi}{a}x\right)e^{-j\bar{\beta}z}e^{j\omega t}$$

First, the power losses are computed, from the available field expressions. The surface current on each plate is

$$K_y = \left|H_z\right|_{x=0,a} = \frac{m\pi C_1}{\omega\mu a}$$

The loss in each plate then becomes

$$\frac{1}{2}K_y^2R_s = \frac{m^2\pi^2C_1^2\sqrt{\omega\mu_m/2\sigma_m}}{2\omega^2\mu^2a^2} \qquad (12.79)$$

The total loss is twice that given by the above expression. The power transmitted in the z-direction per unit area, using Poynting theorem, can be calculated as

$$\frac{1}{2}\text{Re}\left(\mathbf{E}\times\mathbf{H}^*\right)\bullet\hat{\mathbf{z}} = -\frac{1}{2}\left(E_yH_x\right) = \frac{\bar{\beta}C_1^2}{2\omega\mu}\sin^2\left(\frac{m\pi}{a}x\right)$$

The power transmitted in the z- direction per one meter width, when the spacing is a , is

$$\int_{x=0}^{x=a}\frac{\bar{\beta}C_1^2}{2\omega\mu}\sin^2\left(\frac{m\pi}{a}x\right)dx = \frac{\bar{\beta}C_1^2a}{2\omega\mu} \qquad (12.80)$$

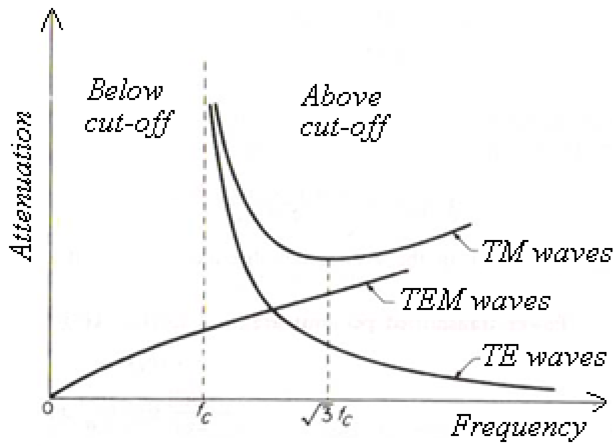


Figure 12.14 Variation of attenuation with frequency in parallel plate system.

With availability of power loss and power flow, the attenuation factor can now be computed, using Eq.(12.77), as

$$\alpha = \frac{2m^2\pi^2\sqrt{\omega\mu_m/2\sigma_m}}{\bar{\beta}\omega\mu a^3} \tag{12.81}$$

Substituting the expression for phase shift constant, from Eq. (12.50) and (12.13), it can be obtained that

$$\alpha = \frac{2m^2\pi^2\sqrt{\omega\mu_m/2\sigma_m}}{\omega\mu a^3\sqrt{\omega^2\mu\varepsilon-\left(m\pi/a^2\right)}} \tag{12.82}$$

From the above expression, it can be observed that the attenuation is infinity at cutoff and falls to low values at higher frequencies. At frequencies very much higher than cutoff, the attenuation varies, inversely as the three halves power of the frequency.

For TM, waves, the attenuation factor can similarly be found. For these waves, the attenuation reaches a minimum at a frequency that is √3 times the cutoff frequency and then increases with frequency. At frequencies much higher than cutoff, the attenuation varies, directly as square root of the frequency. The variation of attenuation with frequency in parallel plate system for different types of waves is shown in Figure 12.14.

UNIT-IV

GUIDED WAVES

Assignment-Cum-Tutorial Questions

SECTION-A

1. Wave in guide travels through [ ]
   (a) Guide walls (b) dielectric (c) Both (a)and (b) (d) None
2. The propagation constant pure real implies [ ]
   (a) Wave without attenuation (b) No wave motion
   (c) Wave with attenuation (d) None
3. The propagation constant pure imaginary implies [ ]
   (a) Wave without attenuation (b) No wave motion
   (c) Wave with attenuation (d) None
4. The propagation constant complex implies [ ]
   (a) Wave without attenuation (b) No wave motion
   (c) Wave with attenuation (d) None
5. In *TM* wave, H can have component [ ]
   a) Parallel to propagation b) Normal to propagation
   c) Both (a) and (b) d) None of these
6. In *TE* wave, *H* can have component [ ]
   a) Parallel to propagation b) Normal to propagation
   c) Both (a) and (b) d) None of these
7. The lowest order *TE* wave in between parallel conducting plates is [ ]
   (a) TE_{10} (b) Principal wave (c) Both(a) and (b) (d) None
8. The principal wave is [ ]
   (a) TM_{00} (b) TEM wave (c) Both(a) and (b). (d) None
9. The nature of the wave normal to plates is [ ]
   (a) Pure standing (b) Pure traveling (c) Impure traveling (d) None
10. The nature of the wave parallel to plates is [ ]
    (a) Pure standing (b) Pure traveling (c) Impure traveling (d) None
11. The following wave in the parallel plate waveguide is called principal wave [ ]
    A) TE wave B) TM wave C) TEM wave D) Plane wave
12. The following wave is called TE wave [ ]
    a. Electric field component normal to wave propagation direction is zero
    b. Electric field component tangential to wave propagation direction is zero
    c. Magnetic field component normal to wave propagation direction is zero
    d. Magnetic field component tangential to wave propagation direction is zero
13. The following mode is not possible in parallel plate waveguide. [ ]
    A) TE_1 B) TE_0 C) TM_1 D) TM_0

14. On the surface of a perfect conductor, the following statement is true [ ]

   a. Tangential component of H field is zero
   b. Normal component of E field is zero
   c. Tangential component of E field is zero
   d. Both tangential and normal of E field is zero

15. Which of the following are not guided waves? [ ]

   A) Waves along ordinary parallel wires B) Waves in waveguide
   C) Waves in co-axial transmission line D) waves travelling in free space

16. The cut-off wavelength of wave in between parallel plane conductors is ————————
17. The phase shift constant for wave in between parallel plane conductors is ———————
18. The phase shift constant for wave in between parallel plane conductors at cut-off is ——
19. The phase shift constant for wave in between parallel plane conductors at high frequencies is ————————————
20. Transmissions lines carry waves in—————— mode where as waveguides carry in——————————————— TEM mode.
21. Pure real value of propagation constant indicates——————— attenuation and ————————————— wave motion.
22. Pure imaginary value of propagation constant indicates————— attenuation and —————————— wave motion.
23. In TE wave the electric vector is——— transverse to the direction of propagation of wave.
24. In TE wave the magnetic vector has——————— component along the direction of propagation of wave.
25. In TM wave the magnetic vector is entirely——— to the direction of propagation of wave.
26. In TM wave the electric vector has a——— along the direction of propagation of wave.
27. In TEM wave both the electric and magnetic vectors are entirely——— the direction of propagation of wave.
28. In mixed or hybrid wave, both the electric and the magnetic vector have components——————————— the direction of propagation of wave.
29. For a perfectly conducting planes $E_{tangential}$ =___________________
30. The cut-off frequency of wave in between parallel plane conductors is ————————
31. A waveguide acts as a _________________ filter.
32. Transverse electric waves are called as _____________ waves.
33. Define degenerative modes?
34. Define Dominant mode.
35. Define Attenuation (α) in parallel plane waveguide.

SECTION-B

Descriptive questions

1. Derive the expressions for field components of TM wave parallel plane waveguide. **[C05]**

2. Derive the expressions for field components of TE wave in parallel plane waveguide. **[C05]**
3. Prove that TEM mode is not possible in parallel plane waveguides. **[C05]**
4. Derive the expressions for the following parameters of TE wave in parallel plane waveguide.
   i) Cutoff frequency
   ii) Phase velocity
   iii) Free space wavelength
   iv) Guided wavelength
   v) Group velocity
   vi) Transverse electric impedance. **[C05]**
5. Derive the expressions for the following parameters of TM wave in parallel plane waveguide. **[C05]**
   i) Cutoff frequency
   ii) Group velocity
   iii) Cutoff wavelength
   iv) Guided wavelength
   v) Transverse magnetic impedance
   vi) Phase velocity
6. Compare the characteristics of TE waves and TM waves in parallel plane waveguide. **[C04]**
7. Prove that the infinite parallel plane conductors act as high pass filter. Define the terms Cut off frequency and cutoff wavelength. **[C04]**
8. Derive TE mode field expressions for guided waves between parallel plates. **[C05]**

Problems

1. A parallel plate waveguide is having a dielectric medium with ε_r =2.25 and μ_r =1. Determine its spacing *a* when its dominant mode cutoff frequency is 5GHz. **[C06]**

2. A parallel plate waveguide of spacing *a*= 4cm, is having a dielectric medium with ε_r =4 and μ_r =1. Determine the TE modes that can propagate when the frequency is 5GHz. Also find f_c and λ_g for each propagating mode. **[C06]**

3. A parallel plate waveguide of spacing *a*= 5cm, is having free space medium in between. If it is excited with fundamental 2GHz and its harmonics, determine all the frequencies that propagate in TE_{10} mode. **[C06]**

4. A 4GHz wave is propagating in a nonmagnetic medium having a dielectric constant, ε_r =2.2. When the phase shift constant is found as 54^0 /cm, find the cutoff wave number. **[C06]**

5. Find the cutoff frequency of TM_2 mode in an air filled parallel plane waveguide. The spacing between the plates is given as 10 cm. **[C06]**

6. Find the phase velocity of a mode propagating at 6 GHz in an air filled parallel plane waveguide. The cutoff frequency of the mode is given as 1.5 GHz. **[C06]**

7. An air filled parallel plane waveguide carries TM_2 mode. The height of the waveguide is 20 cm. If the phase velocity of the mode is 1.5c, find the frequency and guided wavelength of the mode. **[C06]**

8. Find the cutoff frequencies of TM_3 and TM_4 modes of an air filled parallel plane waveguide having height 20 cm. **[C06]**

9. An air filled parallel plane waveguide carries TE_2 mode. The height of the waveguide is 10 cm. if the phase velocity of the mode is equal to velocity of light finds the frequency and guided wavelength of the mode. **[C06]**

10. In a parallel plane waveguide, the phase velocity of TE_3 mode is 1.5c. Find the guided wavelength of TM_2 mode inside the waveguide. The waveguide has been filled with a material having dielectric constant 16 and frequency of the wave is 2 GHz. **[C06]**

SECTION-C

1. Attenuation constant in Np/m due to conductor loss is [ ]

a) $\frac{\text{power dissipated/unit length}}{2\times\text{power flow down the guide}}$ b) $\frac{2\times\text{power flow down the guide}}{\text{power dissipated/unit length}}$

c) $\frac{\text{power dissipated/unit length}}{\text{power flow down the guide}}$ c) None of these

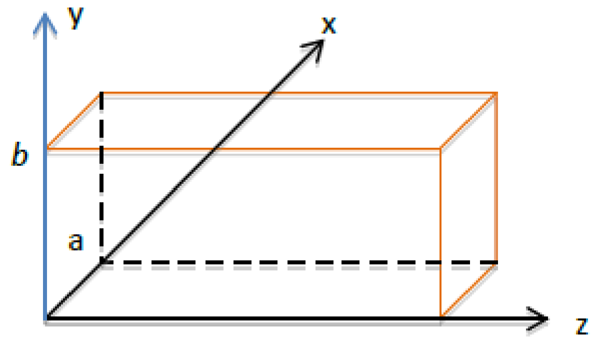
2. Reflective attenuation comes into being when the frequency of the wave is [ ]

a) Less than cut off frequency b) More than cut off frequency

c) Both (a) and (b) d) None of these

UNIT -V
Rectangular Waveguide

we have considered the case of guided wave between a pair of infinite conducting planes. In this lecture, we consider a rectangular wave guide which consists of a hollow pipe of infinite extent but of rectangular cross section of dimension $a \times b$. The long direction will be taken to be the z direction.



Unlike in the previous case $\partial/\partial y$ is not zero in this case. However, as the propagation direction is along the z direction, we have $\frac{\partial}{\partial z} \rightarrow -\gamma$ and $\frac{\partial}{\partial t} \rightarrow i\omega$. We can write the Maxwell's curl equations as

$$\frac{\partial H_z}{\partial y} + \gamma H_y = i\omega\epsilon E_x$$

$$-\gamma H_x - \frac{\partial H_z}{\partial x} = i\omega\epsilon E_y$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = i\omega\epsilon E_z$$

The second set of equations are obtained from the Faraday's law, (these can be written down from above by $E \leftrightarrow H, \epsilon \rightarrow -\mu$

$$\frac{\partial E_z}{\partial y} + \gamma E_y = -i\omega\mu H_x$$

$$-\gamma E_x - \frac{\partial E_z}{\partial x} = -i\omega\mu H_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -i\omega\mu H_z$$

As in the case of parallel plate waveguides, we can express all the field quantities in terms of the derivatives of E_z and H_z. For instance, we have,

$$i\omega\epsilon E_x = \gamma H_y + \frac{\partial H_z}{\partial y}$$

$$= \frac{\gamma}{i\omega\mu}\left(\gamma E_x + \frac{\partial E_z}{\partial x}\right) + \frac{\partial H_z}{\partial y}$$

which gives,

$$\left(i\omega\epsilon - \frac{\gamma^2}{i\omega\mu}\right) E_x = \frac{\gamma}{i\omega\mu}\frac{\partial E_z}{\partial x} + \frac{\partial H_z}{\partial y}$$

so that

$$E_x = -\frac{\gamma}{k^2}\frac{\partial E_z}{\partial x} - \frac{i\omega\mu}{k^2}\frac{\partial H_z}{\partial y} \qquad (1)$$

where

$$k^2 = \gamma^2 + \omega^2\mu\epsilon$$

The other components can be similarly written down,

$$E_y = -\frac{\gamma}{k^2}\frac{\partial E_z}{\partial y} - \frac{i\omega\mu}{k^2}\frac{\partial H_z}{\partial x} \qquad (2)$$

$$H_x = \frac{i\omega\epsilon}{k^2}\frac{\partial E_z}{\partial y} - \frac{\gamma}{k^2}\frac{\partial H_z}{\partial x} \qquad (3)$$

$$H_y = -\frac{i\omega\epsilon}{k^2}\frac{\partial E_z}{\partial x} - \frac{\gamma}{k^2}\frac{\partial H_z}{\partial y} \qquad (4)$$

As before, we will look into the TE mode in detail. Since E_z=0, we need to solve for H_z from the Helmholtz equation,

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2\right) H_z(x,y) = 0 \qquad (5)$$

Remember that the complete solution is obtained by multiplying with $e^{-\gamma z + i\omega t}$. We solve equation (5) using the technique of separation of variables which we came across earlier. Let

$$H_z(x,y) = X(x)Y(y)$$

Substituting this in (5) and dividing by XY throughout, we get,

$$\frac{1}{X}\frac{d^2X}{dx^2} + k^2 = -\frac{1}{Y}\frac{d^2Y}{dy^2} \equiv k_y^2$$

where k_y is a constant. We further define $k_x^2 = k^2 - k_y^2$

We now have two second order equations,

$$\frac{d^2X}{dx^2} + k_x^2 X = 0$$
$$\frac{d^2Y}{dy^2} + k_y^2 Y = 0$$

The solutions of these equations are well known

$$X(x) = C_1 \cos k_x x + C_2 \sin\ k_x x$$
$$Y(y) = C_3 \cos k_y y + C_4 \sin k_y y$$

This gives,

$$H_z(x,y) = C_1 C_3 \cos k_x x \cos k_y y + C_1 C_4 \cos k_x x \sin\ k_y y$$
$$+C_2 C_3 \sin k_x x \cos k_y y + C_2 C_4\ sin\ k_x x \sin k_y y$$

The boundary conditions that must be satisfied to determine the constants is the vanishing of the tangential component of the electric field on the plates. In this case, we have two pairs of plates. The tangential direction on the plates at x=0 and x=aa is the y direction, so that the y component of the electric field

E_y=0 at x=0,.

Likewise, on the plates at y=0 and y=b,

E_x=0 at y=0,

We need first to evaluate E_x and E_y using equations (1) and (2) and then substitute the boundary conditions. Since E_z=0, we can write eqn. (1) and (2) as

$$E_x = -\frac{i\omega\mu}{k^2}\frac{\partial H_z}{\partial y}$$
$$E_y = -\frac{i\omega\mu}{k^2}\frac{\partial H_z}{\partial x}$$

$$E_x = -\frac{i\omega\mu}{k^2}[-C_1 C_3\ \mathrm{k_y}\ \cos\ k_x x \sin k_y y + C_1 C_4\ k_y \cos k_x x\ \cos\ k_y y$$
$$-C_2 C_3\ \mathrm{k_y} \sin k_x x \sin k_y y + C_2 C_4 k_y\ \sin k_x x \cos k_y y$$

$$E_y = -\frac{i\omega\mu}{k^2}[-C_1 C_3\ \mathrm{k_x}\ \sin\ k_x x \cos k_y y - C_1 C_4\ k_x \sin k_x x \sin\ k_y y$$
$$+C_2 C_3\ \mathrm{k_x} \cos k_x x \cos k_y y + C_2 C_4 k_x\ cos\ k_x x \sin k_y y]$$

Since $E_y = 0$ at $x = 0$, we must have $C_2 = 0$ and then we get,

$$E_y = -\frac{i\omega\mu}{k^2}[-C_1 C_3\ \mathrm{k_x}\ \sin\ k_x x \cos k_y y - C_1 C_4\ k_x \sin k_x x \sin\ k_y y]$$

Further, since $E_x = 0$ at y=0, we have $C_4 = 0$. Combining these, we get, on defining a constant $C = C_1 C_3$

$$E_x = \frac{i\omega\mu}{k^2} C k_y \cos k_x x \sin k_y y$$
$$E_y = \frac{i\omega\mu}{k^2} C k_x \sin k_x x \cos k_y y$$

and

$$H_z = C \cos k_x x \cos k_y y$$

We still have the boundary conditions, $E_x = 0$ at $y = b$ and $E_y = 0$ at $x = a$to be satisfied. The former gives $k_y = \frac{n\pi}{b}$ while the latter gives $k_x = \frac{m\pi}{a}$, where m and n are integers. Thus we have,

$$H_z = C \cos\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi y}{b}\right)$$

and

$$k^2 = k_x^2 + k_y^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

However,$k^2 = \omega^2\mu\epsilon + \gamma^2$, so that,

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon}$$

For propagation to take place, γ must be imaginary, so that the cutoff frequency below which propagation does not take place is given by

$$\omega_c = \frac{1}{\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

The minimum cutoff is for $TE_{1,0}$ (or $TE_{0,1}$) mode which are known as *dominant mode*. For these modes E_x (or E_y) is zero.

TM Modes

We will not be deriving the equations for the TM modes for which $H_z=0$. In this case, the solution for E_z, becomes,

$$E_z = E_{z0} \sin\left(\frac{m\pi}{a}\right) \sin\left(\frac{n\pi}{b}\right)$$

As the solution is in terms of product of sine functions, neither m nor n can be zero in this case. This is why the lowest TE mode is the dominant mode.

For propagating solutions, we have,

$$\beta = \sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} = \sqrt{\mu\epsilon}\sqrt{\omega^2 - \omega_c^2}$$

where,

$$\omega_c = \frac{1}{\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

We have, $\omega^2 = \frac{\beta^2}{\mu\epsilon} + \omega_c^2$. Differentiating both sides, we have,

$$\omega\frac{d\omega}{d\beta} = \frac{1}{\mu\epsilon}\beta$$

The group velocity of the wave is given by

$$v_g = \frac{d\omega}{d\beta} = \frac{\beta}{\omega\mu\epsilon} = \frac{\sqrt{\mu\epsilon}\sqrt{\omega^2 - \omega_c^2}}{\omega\mu\epsilon} = \frac{1}{\sqrt{\mu\epsilon}}\sqrt{1 - \frac{\omega_c^2}{\omega^2}}$$

which is less than the speed of light. The phase velocity, however, is given by

$$v_\phi = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\mu\epsilon}\sqrt{\omega^2 - \omega_c^2}} = \frac{1}{\sqrt{\mu\epsilon}}\frac{1}{\sqrt{1 - \frac{\omega_c^2}{\omega^2}}}$$

It may be noted that $v_\phi v_g = \frac{1}{\mu\epsilon}$, which in vacuum equal the square velocity of light.

For propagating TE mode, we have, from (1) and (4) using E_z=0

$$\frac{E_x}{H_y} = \frac{i\omega\mu}{\gamma} = \frac{\omega\mu}{\beta} = \sqrt{\frac{\mu}{\epsilon}}\frac{1}{\sqrt{1 - \frac{\omega_c^2}{\omega^2}}} \equiv \eta_{TE}$$

where η_{TE} is the characteristic impedance for the TE mode. It is seen that the characteristic impedance is resistive. Likewise,

$$\frac{E_y}{H_x} = -\eta_{TE}$$

$$\boldsymbol{\eta_{TE}} = \frac{\boldsymbol{\eta}}{\sqrt{\mathbf{1} - \left(\frac{\boldsymbol{f_C}}{\boldsymbol{f}}\right)^2}}$$

$$\boldsymbol{\eta_{TM}} = \boldsymbol{\eta}\sqrt{\mathbf{1} - \left(\frac{\boldsymbol{f_C}}{\boldsymbol{f}}\right)^2}$$

Impossibility of TEM mode in Rectangular waveguides

We have seen that in a parallel plate waveguide, a TEM mode for which both the electric and magnetic fields are perpendicular to the direction of propagation, exists. This, however is not true of rectangular wave guide, or for that matter for any hollow conductor wave guide without an inner conductor.

We know that lines of H are closed loops. Since there is no z component of the magnetic field, such loops must lie in the x-y plane. However, a loop in the x-y plane, according to Ampere's law, implies an axial current. If there is no inner conductor, there cannot be a real current. The only other possibility then is a displacement current. However, an axial displacement current requires an axial component of the electric field, which is zero for the TEM mode. Thus TEM mode cannot exist in a hollow conductor. (for the parallel plate waveguides, this restriction does not apply as the field lines close at infinity.)

Evanescent wave or mode

This is defined as a wave TE_{mn} or TM_{mn} in which the operating frequency is less than the cutoff frequency and wave propagation does not take place.

observations regarding TM_{mn} mode :

(1) Similar to that of the parallel plane waveguide the fields exists in the discrete electric and magnetic field pattern called modes of waveguide.

(2) All field components are sinusoidally in x and y directions.

(3) All transverse fields go to zero if either m or n is zero. In other words, both the indices m and n have to be non-zero for existence of the TM mode. That is, TM_{m0} and TM_{0n} modes can not exist. Consequently, the lowest order mode which can exist is TM_{11} mode.

Substituting E_z, we get what is called the dispersion relation for the mode as

$$\beta^2 = \omega^2 \mu \in - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2$$

Observations regarding TE mode :

(1) The fields for the TE modes have similar behaviour to the fields of the TM modes i.e they exist in the form of discrete pattern, they have sinusoidal variations in x and y directions, indices m and n represent number of half cycles of the field amplitudes in x and y direction respectively and so on.

(2) Unlike TM mode both indices m and n need not be non-zero for the existence of the TE mode. However, of both the indices zero makes the magnetic field independent of space and therefore cannot exist. In other words,TE_{00} mode cannot exist but TE_{m0} and TE_{0n} modes can exist.

(3) The lowest order mode for the TE case therefore would be TE_{10} and TE_{01}.

Cutt-off Frequecy of TE and TM mode

For both TMmn TEmn and modes the phase constant is given by

$$\beta = \sqrt{\omega^2 \mu \in - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

For the mode to be travelling β has to be a real quantity. If β becomes imaginary then the fields no more remain
travelling but become exponentially decaying

The frequency at which β changes from real to imaginary is called the cut-off frequency of the mode. At cut-off
frequency therefore β giving

$$\omega_c = \frac{1}{\sqrt{\mu\epsilon}}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{1/2}$$

$$\Rightarrow \qquad f_c = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

As the order of the mode increases the cut-off frequency also increases i.e with increasing frequency there is possibility of existence of higher order mode.
The very first mode that propagates on the rectangular waveguide is TE10 mode and therefore this mode is called the dominant mode of the rectangular waveguide.

UNIT-V

RECTANGULAR WAVE GUIDES

Assignment-Cum-Tutorial Questions

SECTION-A

1. Wave in guide travels through [ ]
   (a) Guide walls (b) dielectric
   (c) Both (a) and (b) (d) None
2. The propagation constant pure real implies [ ]
   (a) Wave without attenuation (b) No wave motion
   (c) Wave with attenuation (d) None
3. The propagation constant pure imaginary implies [ ]
   (a) Wave without attenuation (b) No wave motion
   (c) Wave with attenuation (d) None
4. The propagation constant complex implies [ ]
   (a) Wave without attenuation (b) No wave motion
   (c) Wave with attenuation (d) None
5. In *TM* wave, H can have component
   a) Parallel to propagation b) Normal to propagation
   c) Both (a) and (b) d) None of these
6. The bandwidth of RWG is [ ]
   (a) 1:2 (b) 1:3
   (c) 1:4 (d) None
7. Dominant mode in rectangular wave guide is given by [ ]
   a) TE_{10} b) TE_{11}
   c) TE_{01} d) TE_{12}
8. Dominant mode in circular wave guide is given by [ ]
   a) TE_{10} b) TE_{11}
   c) TE_{01} d) TE_{12}
9. In TE_{20} of RWG, the number of half waves in *x*- direction [ ]
   a) 2 b) 1
   c) 4 d) 0
10. In *TE* wave, *H* can have component [ ]
    a) Parallel to propagation b) Normal to propagation
    c) Both (a) and (b) d) None of these
11. The dimension ratio a/b in RWG is [ ]
    (a) 1 (b) 3 (c) 2 (d) None
12. In TE wave of a rectangular waveguide, H can have component. [ ]
    A) Parallel to propagation B) Normal to propagation
    C) Both A and B D) None of these
13. In TE_{31} mode, the number of half wavelengths along y direction is [ ]
    A) 3 B) 1 C) 4 D) 0

14. Purely imaginary propagation constant implies. [ ]

A) No wave propagation
B) Wave propagates with attenuation
C) Wave propagates without attenuation
D) None of these

15. Cutoff frequency of a rectangular wave guide is given by [ ]

A) $\omega\sqrt{\mu\varepsilon}\sqrt{1-\frac{{f_c}^2}{f^2}}$ B) $\frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\frac{m^2}{a^2}+\frac{n^2}{b^2}}$ C) $\frac{\lambda}{\sqrt{1-\frac{{f_c}^2}{f^2}}}$ D) $\frac{\eta}{\sqrt{1-\frac{{f_c}^2}{f^2}}}$

16. Phase constant of a waveguide is given by [ ]

A) $\omega\sqrt{\mu\varepsilon}\sqrt{1-\frac{{f_c}^2}{f^2}}$ B) $\frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\frac{m^2}{a^2}+\frac{n^2}{b^2}}$ C) $\frac{\lambda}{\sqrt{1-\frac{{f_c}^2}{f^2}}}$ D) $\frac{\eta}{\sqrt{1-\frac{{f_c}^2}{f^2}}}$

17. Guided wavelength is given by [ ]

A) $\omega\sqrt{\mu\varepsilon}\sqrt{1-\frac{{f_c}^2}{f^2}}$ B) $\frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\frac{m^2}{a^2}+\frac{n^2}{b^2}}$ C) $\frac{\lambda}{\sqrt{1-\frac{{f_c}^2}{f^2}}}$ D) $\frac{\eta}{\sqrt{1-\frac{{f_c}^2}{f^2}}}$

18. Characteristic wave impedance of the rectangular waveguide in TE mode is given by [ ]

A) $\omega\sqrt{\mu\varepsilon}\sqrt{1-\frac{{f_c}^2}{f^2}}$ B) $\frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\frac{m^2}{a^2}+\frac{n^2}{b^2}}$ C) $\frac{\lambda}{\sqrt{1-\frac{{f_c}^2}{f^2}}}$ D) $\frac{\eta}{\sqrt{1-\frac{{f_c}^2}{f^2}}}$

19. In TM wave the magnetic vector is entirely— to the direction of propagation of wave.
20. In TM wave the electric vector has a—— along the direction of propagation of wave.
21. The cut-off frequency of rectangular waveguide is————
22. The cut-off wavelength of rectangular waveguide is————, its guide wavelength is ——
23. Mode subscripts *m* and *n* in rectangular waveguides indicate the no. of half wavelengths along—— directions respectively.
24. Dominant mode in rectangular guides is—— whereas in circular waveguides it is —
25. Cut-off wavelength for dominant mode is equal to 2*a* where *a* is —— distance between the sidewalls of the waveguide.
26. Degenerate modes of waveguides are—— modes having same cut-off frequency.
27. Rectangular waveguides are dimensioned with ratio *a*/*b* approximately equal to——.
28. In a TE mode _____________ [ ]

A) $E_z = 0$ B) $H_z = 0$ C) $E_z = H_z = 0$ D) $f_c = 0$

29. TEM mode propagation is not possible in the following transmission medium. [ ]

A) Co-axial cable B) Two wire transmission line

C) Micro strip lines D) Rectangular waveguide

30. The cut-off wavelength λ_c for a dominant mode in rectangular waveguide ___

31. The mode having the lowest cut-off frequency in rectangular waveguide is ____

32. For an air filled rectangular waveguide, the guided wavelength λ_g is ______

33. In a **TE** $_{\mathbf{m\,n}}$ mode of a rectangular waveguide, ***m*** and ***n*** represents_____

34. The Dominant mode in the rectangular waveguide is _____________

35. Cutoff wavelength of a rectangular waveguide in TE_{10} mode is ___________

36. When the broader wall dimension of the rectangular waveguide a = 2.4 cm, find the cutoff wavelength for dominant mode λ_c = _____

37. Guided wavelength (λ_g) for an air filled waveguide is 8.4 cm. If the same waveguide is filled with dielectric material of having ϵ_r = 4, new λ_g _____

38. What is TE, TM waves in rectangular waveguides?

39. Give example of a pair of degenerative modes in rectangular waveguide and justify your answer.

SECTION-B

Descriptive questions

1. Define and differentiate phase velocity from group velocity. **[C04]**
2. Prove that the velocity of the wave in waveguides is a function of frequency. What is guide wavelength? **[C04]**
3. Prove that the rectangular waveguide is high pass filter. Define the terms cut-off frequency and cut-off wavelength. **[C04]**
4. What is the meaning of mode of the wave? Give a complete description of modal propagation characteristics of waves in RWGs. **[C04]**
5. How the bandwidth of waveguides is defined? How the dimensions of the rectangular waveguides are selected to give maximum bandwidth. **[C04]**
6. Derive TE mode field expressions for rectangular waveguide. **[C05]**
7. Derive the expressions for field components of TE wave in rectangular waveguide. **[C05]**
8. Illustrate the field patterns of TE_{10} and TE_{20} modes in rectangular waveguides with neat sketches. **[C05]**
9. Explain why TEM mode is not possible in rectangular waveguide. **[C05]**
10. State the formulas for the following parameters related to rectangular waveguide operated in TE_{mn} mode:

    i. Cutoff frequency

ii. Cutoff wave-number
iii. Propagation constant
iv. Wavelength in the waveguide
v. Phase constant
vi. Phase velocity

11. Derive the expressions for field components of TM wave in rectangular waveguide. **[C05]**
12. Explain the following: i) Dominant mode, ii) Degenerative modes. **[C04]**

Problems

1. The dimensions of an air dielectric waveguide working at 5.2GHz are 4.75×2.21 cm. Find its (a) dominant mode cutoff frequency and (b) guide wavelength. **[C06]**
   Answers: (a) 3.157 GHz (b) 7.26cm
2. The ratio of dimensions of an air dielectric waveguide are $a/b=2$. Its dominant mode cutoff frequency is 850MHz and guide wavelength is 40cm. Find its (a) operating frequency, (b) dimensions of guide and (b) phase shift constant. **[C06]**
   Answers: (a) 1.13 GHz (b) 17.65 × 8.82cm (c) 15.59 rad/m
3. The ratio of dimensions of an air dielectric waveguide are $a/b=2$. Its dominant mode cutoff frequency is 9GHz. When it is designed to work in 12.5 to 19GHz range, find its dimensions. **[C06]**
   Answers: 1.66 × 0.83cm
4. A loss-less air-dielectric S-band waveguide, carrying wave in TE_{11} mode, has inside dimensions 7.214×3.404cm. When the operating frequency is 1.2 times the cutoff frequency of the mode, find (a) cutoff wave number (b) cutoff frequency (c) operating frequency (d) propagation constant (e) cutoff wavelength (f) operating wavelength and (g) guide wavelength . **[C06]**
   Answers: 102.05 rad/m, 4.87GHz, 5.84GHz, j67.61/m, 6.16cm, 5.14cm, 10.10cm
5. The dimensions of an air filled rectangular waveguide are 4 cm x 2 cm. Find (i) Cutoff frequency for the dominant mode (ii) guide wavelength at 6 GHz. **[C06]**
6. The ratio of dimensions of an air filled rectangular waveguide is a/b = 2. Its cutoff wavelength is 3.32cm. (i) Find the dimensions of the waveguide (ii) Phase shift constant at 12 GHz. **[C06]**
7. An air filled rectangular wave guide has dimensions of a = 7 cm and b = 3.5 operates in the dominant mode. **[C06]**
   i. Cutoff frequency
   ii. Phase velocity of the wave in the guide at a frequency of 3.5 GHz.
   iii. Guided wavelength at the same frequency.

8. Consider a rectangular waveguide with dimensions a = 2.29 cm and b = 1.145 cm is filled with dielectric material having ε_r = 2.5 and μ_r =1. The cutoff wavelength and cutoff frequency for TE_{10}, TE_{20} and TM_{11}. **[C06]**
9. Find out the cutoff frequencies for TM_{12} mode in air filled rectangular waveguide whose dimensions ***a*** and ***b***, respectively are (i) 10. x 0.5 cm, (ii) 0.5 x 1.0 cm, (iii) 1.0 x 1.0 cm. **[C06]**
10. A standard rectangular waveguide has internal dimensions a = 1.905 cm and, b = 0.953 cm. the waveguide is air filled and propagates waves at 18 GHz. Calculate the following parameters for the TM_{11} mode: **[C06]**

i) Critical frequency
ii) Guide wavelength
iii) Phase constant
iv) Propagation Constant
v) Phase velocity
vi) Wave impedance

SECTION-C

1. The phase velocity $\bar{v}$ is given by [ ]

   (a) $\bar{v} = \omega/\beta$ (b) $\bar{v} = \omega\Big/\sqrt{\omega^2\mu\varepsilon - (m\pi/a)^2}$

   (c) $\bar{v} = 1\Big/\sqrt{\mu\varepsilon}\sqrt{1+(\lambda/\lambda_c)^2}$ (d) All

2. The cut off frequency of TEM wave is [ ]

   (a) Infinite (b) Zero (c) $1/2\pi\sqrt{\mu\varepsilon}$ (d) None

3. The ratio of **E** to **H** is free space intrinsic impedance only in case of [ ]

   (a) TEM wave (b) TE wave (c) TM wave (d) None

4. The group velocity v_g is given by [ ]

   (a) $v_g = c^2/\bar{v}$ (b) $v_g = c\sqrt{1-(m\lambda/2a)^2}$

   (c) Both(a) and (b) (d) None

circular wave guides:

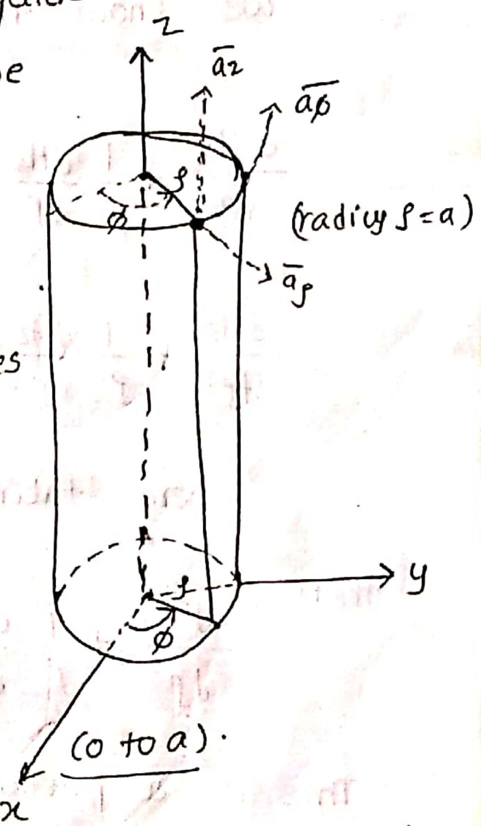
A circular wave guide is basically a tubular, circular conductor. it is a hollow metalic tube with circular cross section.

Manufacturing of circular wave guide is easy compared to that of the rectangular wave guide.

circular waveguides are easily connected to each other.

It converts linearly polarized waves into circular polarized wave.

The figure shows a circular wave guide of radius $\rho=a$ and length along z-direction, ϕ varies from 0 to 2π and ρ varies from (0 to a).



Solution of wave Equation in cylindrical co-ordinates:

propagation of TE waves in circular wave guides:

For TE wave propagation $E_z=0$, & $H_z\neq 0$

from Maxwell's wave equation

we know that

$$\nabla^2 H = \gamma^2 H \quad \text{[helmholz equation]}$$

$$\gamma = \sqrt{j\omega\mu(\sigma+j\omega\varepsilon)} \quad [\sigma=0]$$

$$\gamma^2 = -\omega^2\mu\varepsilon$$

here $H_z\neq 0$ [wave propagation along z-direction]

$$\nabla^2 H_z = -\omega^2\mu\varepsilon H_z \longrightarrow (1)$$

Expanding in cylindrical co-ordinates.

$$\frac{d^2 H_z}{d\rho^2} + \frac{1}{\rho}\frac{dH_z}{d\rho} + \frac{1}{\rho^2}\frac{d^2 H_z}{d\phi^2} + \frac{d^2 H_z}{dz^2} = -\omega^2 \mu \epsilon H_z .$$

we know that $\frac{d^2}{dz^2} = \gamma^2$ [an operator]
[propagation of wave along z direction is constant]

$$\frac{d^2 H_z}{d\rho^2} + \frac{1}{\rho}\frac{dH_z}{d\rho} + \frac{1}{\rho^2}\frac{d^2 H_z}{d\phi^2} + \gamma^2 H_z = -\omega^2 \mu \epsilon H_z .$$

$$\frac{d^2 H_z}{d\rho^2} + \frac{1}{\rho}\frac{dH_z}{d\rho} + \frac{1}{\rho^2}\frac{d^2 H_z}{d\phi^2} + (\gamma^2 + \omega^2 \mu \epsilon) H_z = 0.$$

here taking $\gamma^2 + \omega^2 \mu \epsilon = h^2$

then

$$\frac{d^2 H_z}{d\rho^2} + \frac{1}{\rho}\frac{dH_z}{d\rho} + \frac{1}{\rho^2}\frac{d^2 H_z}{d\phi^2} + h^2 H_z = 0 \rightarrow (2)$$

This is a partial differential equation, whose solution by separable of variables method is assumed to be

$$H_z = R\,Q$$

where R = function of 'ρ' only
Q = function of 'ϕ' only.

$$\frac{d^2 (RQ)}{d\rho^2} + \frac{1}{\rho}\frac{d(RQ)}{d\rho} + \frac{1}{\rho^2}\frac{d^2 (RQ)}{d\phi^2} + h^2 (RQ) = 0$$

$$Q\frac{d^2 R}{d\rho^2} + \frac{Q}{\rho}\frac{dR}{d\rho} + \frac{R}{\rho^2}\frac{d^2 Q}{d\phi^2} + h^2 (RQ) = 0.$$

multiply with '$\frac{\rho^2}{RQ}$'. On both sides

$$\underset{(1)}{\frac{\rho^2}{R}\cdot\frac{d^2 R}{d\rho^2}} + \underset{(2)}{\frac{\rho}{R}\frac{dR}{d\rho}} + \underset{(3)}{\frac{1}{Q}\frac{d^2 Q}{d\phi^2}} + \underset{(4)}{\rho^2 h^2} = 0 . \rightarrow (3)$$

In above equation ③, the terms ①, ② & ④ are functions of ρ only.

and the term ③ is function of ϕ only.

$\therefore$ let $\frac{1}{\Phi}\frac{d^2\Phi}{d^2\phi} = -n^2$, where n^2 is constant.

then $\frac{d^2\Phi}{d\phi^2} = -n^2\Phi$.

$$\frac{d^2\Phi}{d\phi^2} + n^2\Phi = 0$$

The Solution is $\Phi = A_n \cos n\phi + B_n \sin n\phi \rightarrow$ ④.

and eq ③ can be reduced as.

$$\frac{\rho^2}{R}\frac{d^2R}{d\rho^2} + \frac{\rho}{R}\frac{dR}{d\rho} + (\rho^2h^2 - n^2) = 0$$

Multiplying throught by R.

$$\rho^2\frac{d^2R}{d\rho^2} + \rho\frac{dR}{d\rho} + (\rho^2h^2 - n^2)R = 0 \rightarrow ⑤$$

This is similar to Bessel's equation of the form

$$x^2\frac{d^2y}{dx^2} + x\cdot\frac{dy}{dx} + (x^2 - n^2)y = 0 \rightarrow ⑥$$

The Solution for this equation is

$$y = C_n \cdot J_n(x)$$

where $J_n(x)$ represents n^{th} order

now bring eq ⑤ in the form of eq ⑥

$$(\rho h)^2 \frac{d^2R}{d(\rho h)^2} + \rho h \frac{dR}{d(\rho h)} + (\rho^2 h^2 - n^2) R = 0$$

$[\because d(2x) = 2dx]$

$\therefore$, the solution of this equation is :

$$R = C_n J_n(\rho h) \rightarrow (7)$$

from equations (4) & (7)

$$H_z = RQ$$

$$H_z = C_n J_n(\rho h) . (A_n \cos n\phi + B_n \sin n\phi)$$

The n^{th} order bessel function $J_n(\rho h)$ are plotted for different values of n.

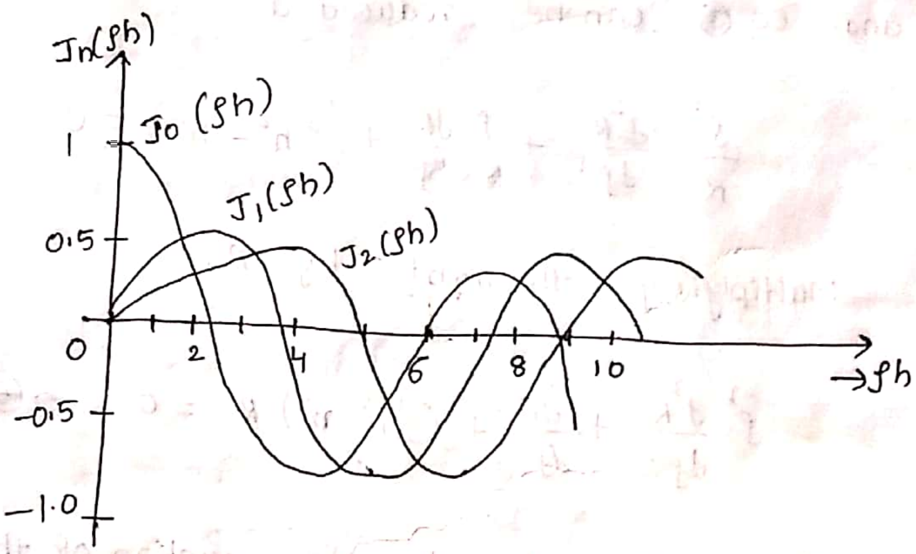


fig: Bessel function of first kind.

$\therefore$ The complete solution becomes as per eq

$$H_z = RQ.$$

$$H_z = C_n J_n(\rho h) . \sqrt{A_n^2 + B_n^2} \cos\left[n\phi + \tan^{-1}\left(\frac{A_n}{B_n}\right)\right]$$

$$H_z = C_n J_n(\rho h) . C_n' \cos n\phi$$

where $C_n' = \sqrt{A_n^2 + B_n^2}$ $\quad n\phi = n\phi + \tan^{-1}\left(\frac{A_n}{B_n}\right)$

let $C_n C_n' = C_0$ (another constant)

then $$H_z = C_0 J_n(\rho h) \cos n\phi \, e^{-\gamma z} \rightarrow (9)$$

if we consider a sinusoidal variation along 'z'.

for lossless dielectric maxwell's equations are.

$$\nabla \times \bar{H} = j\omega\varepsilon \bar{E} \quad \& \quad \nabla \times \bar{E} = -j\omega\mu H.$$

Expanding in cylindrical co-ordinates.

$$\nabla \times E = \begin{vmatrix} \bar{a}_\rho & \bar{a}_\phi & \bar{a}_z \\ i & j & k \\ \frac{d}{d\rho} & \frac{d}{\rho d\phi} & \frac{d}{dz} \\ E_\rho & E_\phi & E_z \end{vmatrix} = -j\omega\mu H$$

$$= \frac{1}{\rho} \begin{vmatrix} i & \rho j & k \\ \frac{d}{d\rho} & \frac{d}{d\phi} & \frac{d}{dz} \\ E_\rho & \rho E_\phi & E_z \end{vmatrix} = -j\omega\mu[H_\rho i + H_\phi j + H_z k]$$

$$\Rightarrow \frac{1}{\rho}\left[i\left(\frac{dE_z}{d\phi} - \frac{\rho dE_\phi}{dz}\right) - \rho j\left(\frac{dE_z}{d\rho} - \frac{dE_\rho}{dz}\right) + k\left(\frac{d\rho E_\phi}{d\rho} - \frac{dE_\rho}{d\phi}\right)\right]$$

$$= -j\omega\mu[H_\rho i + H_\phi j + H_z k]$$

By compare on both sides

$$\frac{1}{\rho}\frac{dE_z}{d\phi} - \frac{dE_\phi}{dz} = -j\omega\mu H_\rho \rightarrow (10)$$

$$-\frac{dE_z}{d\rho} + \frac{dE_\rho}{dz} = -j\omega\mu H_\phi \rightarrow (11)$$

$$\frac{1}{\rho}\frac{d(\rho E_\phi)}{d\rho} - \frac{1}{\rho}\frac{dE_\rho}{d\phi} = -j\omega\mu H_z \rightarrow (12)$$

Similarlly

$$\nabla \times H = \frac{1}{\rho}\begin{vmatrix} i & \rho j & k \\ \frac{d}{d\rho} & \frac{d}{d\phi} & \frac{d}{dz} \\ H_\rho & \rho H_\phi & H_z \end{vmatrix} = j\omega\varepsilon\left[E_\rho \bar{i} + E_\phi \bar{j} + E_z \bar{k}\right]$$

$$\Rightarrow \frac{1}{\rho}\left[i\left[\frac{dH_z}{d\phi} - \rho\frac{dH_\phi}{dz}\right] - \rho j\left[\frac{dH_z}{d\rho} - \frac{dH_\rho}{dz}\right] + \bar{k}\left[\frac{d(\rho H_\phi)}{d\rho} - \frac{dH_r}{d\phi}\right]\right]$$

$$= j\omega\varepsilon\left[E_\rho i + E_\phi \bar{j} + E_z \bar{k}\right]$$

By comparing on both sides.

$$\frac{1}{\rho}\frac{dH_z}{d\phi} - \frac{dH_\phi}{dz} = j\omega\varepsilon E_\rho \longrightarrow (13)$$

$$\frac{dH_\rho}{dz} - \frac{dH_z}{d\rho} = j\omega\varepsilon E_\phi \longrightarrow (14)$$

$$\frac{1}{\rho}\frac{d(H_\phi)}{d\rho} - \frac{1}{\rho}\frac{dH_\rho}{d\phi} = j\omega\varepsilon E_z \longrightarrow (15)$$

By substituting $\frac{d}{dz} = -\gamma$ and $E_z = 0$ in above equations. i.e for TE wave.

$$\gamma = j\beta \quad [\alpha = 0]$$

$$\boxed{\frac{d}{dz} = -\gamma = -j\beta} \qquad \boxed{E_z = 0}$$

then

$$-\frac{dE_\phi}{dz} = -j\omega\mu H_\rho \rightarrow (10)$$

~~$(-j\beta E_\rho) = +j\omega\mu H_\phi$~~

~~$j\beta E_\rho = j\omega\mu H_\phi$~~

~~$\beta E_\rho = \omega\mu H_\phi$~~ $\rightarrow$ (11)

from above ⑩ $-\frac{\partial E_\phi}{\partial z} = -j\omega\mu H_\rho$

$$\gamma E_\phi = -j\omega\mu H_\rho \rightarrow ⑩ \rightarrow (a)$$

~~$\gamma E_\rho = j\omega\mu H_\phi$~~

and $-\frac{\partial E_\rho}{\partial z} = j\omega\mu H_\phi$

$$\gamma E_\rho = j\omega\mu H_\phi \rightarrow ⑪ \rightarrow (b)$$

$$\frac{1}{\rho}\frac{\partial(\rho E_\phi)}{\partial \rho} - \frac{1}{\rho}\frac{\partial E_\rho}{\partial \phi} = -j\omega\mu H_z \rightarrow ⑫ \rightarrow (c)$$

from eq's ⑬ to ⑮.

$$\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} + \gamma H_\phi = j\omega\varepsilon E_\rho \rightarrow ⑬ \rightarrow (d).$$

$$-\gamma H_\rho - \frac{\partial H_z}{\partial \rho} = j\omega\varepsilon E_\phi$$

$$\frac{\partial H_z}{\partial \rho} + \gamma H_\rho = -j\omega\varepsilon E_\phi \rightarrow ⑭ \rightarrow (e).$$

$$\frac{1}{\rho}\frac{\partial(\rho H_\phi)}{\partial \rho} - \frac{1}{\rho}\frac{\partial H_r}{\partial \phi} = 0 \rightarrow ⑮ \rightarrow (f).$$

$$\frac{\partial(\rho H_\phi)}{\partial \rho} = \frac{\partial H_r}{\partial \phi} \rightarrow (g)$$

from eq (a) $E_\phi = \frac{-j\omega\mu H_\rho}{\gamma}$, $H_\rho = \frac{\gamma E_\phi}{-j\omega\mu}$

substitute in eq (e)

$$\frac{\partial H_z}{\partial \rho} + \gamma H_\rho = -j\omega\varepsilon E_\phi$$

$$\frac{\partial H_z}{\partial \rho} + \gamma H_\rho = -j\omega\varepsilon\left[\frac{-j\omega\mu H_\rho}{\gamma}\right]$$

$$= \left[\frac{-\omega^2\mu\varepsilon}{\gamma}\right] H_\rho$$

$$\frac{dH_z}{d\rho} = \left[\frac{-\omega^2\mu\epsilon}{\gamma}\right] H_\rho - \gamma H_\rho$$

$$= H_\rho \left[\frac{-\omega^2\mu\varepsilon - \gamma^2}{\gamma}\right]$$

$$= H_\rho \left[\frac{-h^2}{\gamma}\right]$$

$$\boxed{H_\rho = \frac{-\gamma}{h^2} \frac{dH_z}{d\rho}} \longrightarrow (16)$$

again from eq (a) substitute H_ρ value.

$$\frac{dH_z}{d\rho} + \gamma \left(\frac{\gamma E_\phi}{-j\omega\mu}\right) = -j\omega\varepsilon E_\phi$$

$$\frac{dH_z}{d\rho} - \frac{\gamma^2}{j\omega\mu} E_\phi = -j\omega\varepsilon E_\phi$$

$$\frac{dH_z}{d\rho} = E_\phi \left[-j\omega\varepsilon + \frac{\gamma^2}{j\omega\mu}\right]$$

$$= E_\phi \left[\frac{\omega^2\mu\varepsilon + \gamma^2}{j\omega\mu}\right]$$

$$= E_\phi \left[\frac{h^2}{j\omega\mu}\right]$$

$$\boxed{E_\phi = \frac{j\omega\mu}{h^2} \frac{dH_z}{d\rho}} \longrightarrow (17)$$

and now from eq (b)

$$E_\rho = \frac{j\omega\mu H_\phi}{\gamma}, \quad H_\phi = \frac{\gamma E_\rho}{j\omega\mu}$$

and from eq (d).

$$\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} + \gamma H_\phi = j\omega\varepsilon E_\rho$$

$$\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} + \gamma H_\phi = j\omega\varepsilon\left[\frac{j\omega\mu H_\phi}{\gamma}\right]$$

$$= \left[\frac{-\omega^2\mu\varepsilon}{\gamma}\right] H_\phi$$

$$\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} = H_\phi\left[\frac{-\omega^2\mu\varepsilon}{\gamma} - \gamma\right]$$

$$= H_\phi\left[\frac{-h^2}{\gamma}\right]$$

$$\boxed{H_\phi = \frac{-\gamma}{h^2}\frac{1}{\rho}\frac{\partial H_z}{\partial \phi}} \longrightarrow (18)$$

from eq (d) $\quad \frac{1}{\rho}\frac{\partial H_z}{\partial \phi} + \gamma\left(\frac{\gamma E_\rho}{j\omega\mu}\right) = j\omega\varepsilon E_\rho$

$$\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} + \frac{\gamma^2}{j\omega\mu}E_\rho = j\omega\varepsilon E_\rho$$

$$\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} = E_\rho\left[j\omega\varepsilon - \frac{\gamma^2}{j\omega\mu}\right]$$

$$= E_\rho\left(\frac{-h^2}{j\omega\mu}\right)$$

$$\boxed{E_\rho = \frac{-j\omega\mu}{h^2}\frac{1}{\rho}\frac{\partial H_z}{\partial \phi}} \rightarrow (19)$$

The boundary conditions require that the ϕ component of the electric field E_ϕ, which is tangential to the inner surface of the circular waveguide at $\rho = a$, must vanish or that the ρ component of the magnetic field H_ρ, which is normal to the inner surface $\rho = a$, must vanish.

consequently

$$E_\phi = 0 \quad , \quad \rho = a.$$

now From eq (17) $\quad E_\phi = \frac{j\omega\mu}{h^2} \frac{dH_z}{d\rho}$

i.e $\quad 0 = \frac{dH_z}{d\rho}$

$$\left.\frac{dH_z}{d\rho}\right|_{\rho=a} = 0.$$

we know that Solution of H_z from eq (9)

$$H_z = C_0 J_n(\rho h) \cos n\phi \, e^{-\gamma z}$$

$$\left.\frac{dH_z}{d\rho}\right|_{\rho=a} = C_0 J_n'(\rho h) \cdot \cos n\phi \, e^{-\gamma z} = 0 \cdot \rightarrow (20)$$

then $\quad J_n'(ah) = 0.$

Here the J_n' indicates the derivative of J_n.

since the J_n are oscillatory functions, $J_n'(ah)$ are also oscillatory functions. An infinite sequence of values of (ah) satisfies eq (20). These points, the roots of eq (20) correspond to the maxima and minima of the curves $J_n'(ah)$.

let P'_{nm} be the possible values of root of eq (20).

then $ah = P'_{nm}$

$$h = \frac{P'_{nm}}{a}$$

P'_{nm} values for TE_{nm} mode in circular waveguide.

n ↓ \ m →	1	2	3
0	3.832	7.016	10.173
1	1.841	5.331	8.536
2	3.054	6.706	9.969
3	4.201	8.015	11.346

∴ TE_{nm} modes in circular wave guides are

$$H_z = C_0 \cdot J_n\left(\frac{P'_{nm}}{a}\rho\right) \cos n\phi \, e^{-\gamma z} \rightarrow (21)$$

from eq (16)

$$H_\rho = \frac{-\gamma}{h^2}\frac{dH_z}{d\rho} = \frac{-\gamma}{h^2} C_0 J_n'\left(\frac{P'_{nm}}{a}\rho\right) \cos n\phi \, e^{-\gamma z} \rightarrow (22)$$

from eq (17) $$E_\phi = \frac{j\omega\mu}{h^2}\frac{dH_z}{d\rho}$$

$$E_\phi = \frac{j\omega\mu}{h^2} C_0 J_n'\left(\frac{P'_{nm}}{a}\rho\right) \cos n\phi \, e^{-\gamma z} \rightarrow (23)$$

from eq (18) $$H_\phi = \frac{-\gamma}{h^2}\frac{1}{\rho}\frac{dH_z}{d\phi}$$

$$H_\phi = +\frac{\gamma}{h^2}\frac{1}{\rho} C_0 J_n\left(\frac{P'_{nm}}{a}\rho\right) n \sin n\phi \, e^{-\gamma z} \rightarrow (24)$$

from eq (19) $$E_\rho = \frac{-j\omega\mu}{h^2}\frac{1}{\rho} \cdot \frac{dH_z}{d\phi}$$

$$E_\rho = \frac{+j\omega\mu}{h^2}\frac{1}{\rho} C_0 J_n\left(\frac{P'_{nm}}{a}\rho\right) n \sin n\phi \, e^{-\gamma z} \rightarrow (25).$$

TM modes in circular Wave guides:

For TM modes $H_z = 0$, $E_z \neq 0$.

consider the wave equation is

$$\nabla^2 E_z = -\omega^2 \mu \varepsilon E_z$$

[from Helm holtz equation] [$\nabla^2 E = \gamma^2 E$]

The Solution for this equation is given by the variable seperable method.

$$E_z = C_0 J_n(\rho h) \cos n\phi \, e^{-\gamma z} \rightarrow (1)$$

[similar to TE mode solution]

Now consider

a lossless dielectric maxwell's equations are

$$\nabla \times \bar{E} = -j\omega\mu H$$

$$\nabla \times \bar{H} = j\omega\varepsilon E$$

$$\nabla \times E = \frac{1}{\rho} \begin{vmatrix} \bar{i} & \rho\bar{j} & \bar{k} \\ \frac{d}{d\rho} & \frac{d}{d\phi} & \frac{d}{dz} \\ E_\rho & \rho E_\phi & E_z \end{vmatrix} = -j\omega\mu \left[H_\rho \bar{i} + H_\phi \bar{j} + H_z \bar{k} \right]$$

$$\Rightarrow \frac{1}{\rho}\left[i\left(\frac{dE_z}{d\phi} - \rho\frac{dE_\phi}{dz}\right) - \rho\bar{j}\left(\frac{dE_z}{d\rho} - \frac{dE_\rho}{dz}\right) + \bar{k}\left(\frac{d(\rho E_\phi)}{d\rho} - \frac{dE_\rho}{d\phi}\right)\right]$$

$$= -j\omega\mu\left[H_\rho \bar{i} + H_\phi \bar{j} + H_z \bar{k} \right]$$

By comparing on both sides.

$$\frac{1}{\rho} \frac{dE_z}{d\phi} - \frac{dE_\phi}{dz} = -j\omega\mu H_\rho \longrightarrow (2)$$

$$\frac{dE_z}{d\phi} - \frac{dE_\rho}{dz} = j\omega\mu H_\phi \longrightarrow (3)$$

Scanned with CamScanner

$$\frac{1}{\rho}\frac{d(\rho E_\phi)}{d\rho} - \frac{1}{\rho}\frac{dE_\rho}{d\phi} = -j\omega\mu H_z \longrightarrow (4)$$

$$\nabla \times H = j\omega\varepsilon E$$

$$\nabla \times H = \frac{1}{\rho}\begin{vmatrix} i & \rho j & k \\ \frac{d}{d\rho} & \frac{d}{d\phi} & \frac{d}{dz} \\ H_\rho & \rho H_\phi & H_z \end{vmatrix} = j\omega\varepsilon\left[E_\rho \bar{i} + E_\phi \bar{j} + E_z \bar{k}\right]$$

$$\Rightarrow \frac{1}{\rho}\left[i\left(\frac{dH_z}{d\phi} - \rho\frac{dH_\phi}{dz}\right) - \rho j\left(\frac{dH_z}{d\rho} - \frac{dH_\rho}{dz}\right) + k\left(\frac{d(\rho H_\phi)}{d\rho} - \frac{dH_\rho}{d\phi}\right)\right]$$

$$= j\omega\varepsilon\left[E_\rho \bar{i} + E_\phi \bar{j} + E_z \bar{k}\right]$$

on comparing on bothsides

$$\frac{1}{\rho}\frac{dH_z}{d\phi} - \frac{dH_\phi}{dz} = j\omega\varepsilon E_\rho \longrightarrow (5)$$

$$\frac{dH_z}{d\rho} - \frac{dH_\rho}{dz} = -j\omega\varepsilon E_\phi \longrightarrow (6)$$

$$\frac{1}{\rho}\frac{d(\rho H_\phi)}{d\rho} - \frac{1}{\rho}\frac{dH_\rho}{d\phi} = j\omega\varepsilon E_z \longrightarrow (7)$$

for TM waves, $H_z = 0$ & $E_z \neq 0$. $\frac{d}{dz} = -\gamma$

from eq (2) $\frac{1}{\rho}\frac{dE_z}{d\phi} - \frac{dE_\phi}{dz} = -j\omega\mu H_\rho$

$$\frac{1}{\rho}\frac{dE_z}{d\phi} + \gamma E_\phi = -j\omega\mu H_\rho \longrightarrow (a)$$

from eq (3) $\frac{dE_z}{d\rho} - \frac{dE_\rho}{dz} = j\omega\mu H_\phi$

$$\frac{dE_z}{d\rho} + \gamma E_\rho = j\omega\mu H_\phi \longrightarrow (b)$$

from eq (4) $\frac{1}{\rho}\frac{d(\rho E_\phi)}{d\rho} - \frac{1}{\rho}\frac{dE_\rho}{d\phi} = -j\omega\mu H_z$

$$\frac{d(\rho E_\phi)}{d\rho} = \frac{dE_\rho}{d\phi} \longrightarrow (c)$$

from eq (5)

$$0 - \frac{dH_\phi}{dz} = j\omega\varepsilon E_\rho$$

$$\gamma H_\phi = j\omega\varepsilon E_\rho \longrightarrow (d)$$

from eq (6)

$$0 - \frac{dH_\rho}{dz} = -j\omega\varepsilon E_\phi$$

$$\gamma H_\rho = -j\omega\varepsilon E_\phi \longrightarrow (e)$$

forom eq (7)

$$\frac{1}{\rho}\frac{d(\rho H_\phi)}{d\rho} - \frac{1}{\rho}\frac{dH_\rho}{d\phi} = j\omega\varepsilon E_z \rightarrow (f)$$

now from eq (d).

$$H_\phi = \frac{j\omega\varepsilon E_\rho}{\gamma}, \quad E_\rho = \frac{H_\phi \gamma}{j\omega\varepsilon}$$

Substuting in eq (b).

$$\frac{dE_z}{d\rho} + \gamma E_\rho = j\omega\mu H_\phi$$

$$= j\omega\mu\left(\frac{j\omega\varepsilon E_\rho}{\gamma}\right)$$

$$\frac{dE_z}{d\rho} = \left(\frac{-\omega^2\mu\varepsilon}{\gamma} - \gamma\right) E_\rho$$

$$= \left(\frac{-h^2}{\gamma}\right) E_\rho$$

$$\boxed{\therefore E_\rho = \frac{-\gamma}{h^2}\frac{dE_z}{d\rho}} \longrightarrow (8)$$

again eq (b)

$$\frac{dE_z}{d\rho} + \gamma\left[\frac{H_\phi \gamma}{j\omega\varepsilon}\right] = j\omega\mu H_\phi$$

$$\frac{dE_z}{d\rho} = H_\phi\left[j\omega\mu - \frac{\gamma^2}{j\omega\varepsilon}\right]$$

$$= \left[\frac{-\omega^2\mu\varepsilon-\gamma^2}{j\omega\varepsilon}\right] H_\phi$$

$$= \left[\frac{-h^2}{j\omega\varepsilon}\right] H_\phi$$

$$\boxed{H_\phi = \frac{-j\omega\varepsilon}{h^2}\frac{dE_z}{d\rho}} \rightarrow (9)$$

now consider eq (c)

$$H_\rho = \frac{-j\omega\varepsilon E_\phi}{\gamma}, \quad E_\phi = \frac{\gamma H_\rho}{-j\omega\varepsilon}$$

from eq (a)

$$\frac{1}{\rho}\frac{dE_z}{d\phi} + \gamma E_\phi = -j\omega\mu H_\rho$$

$$\frac{1}{\rho}\frac{dE_z}{d\phi} + \gamma E_\phi = -j\omega\mu\left[\frac{-j\omega\varepsilon E_\phi}{\gamma}\right]$$

$$\frac{1}{\rho}\frac{dE_z}{d\phi} = \left[\frac{-\omega^2\mu\varepsilon}{\gamma} - \gamma\right] E_\phi$$

$$\frac{1}{\rho}\frac{dE_z}{d\phi} = \left[\frac{-h^2}{\gamma}\right] E_\phi$$

$$E_\phi = \frac{-\gamma}{h^2\rho}\frac{dE_z}{d\phi} \rightarrow (10)$$

and again

$$\frac{1}{\rho}\frac{dE_z}{d\phi} + \gamma\left[\frac{\gamma H_\rho}{-j\omega\varepsilon}\right] = -j\omega\mu H_\rho$$

$$\frac{1}{\rho}\frac{dE_z}{d\phi} = -j\omega\mu H_\rho + \frac{\gamma^2 H_\rho}{j\omega\varepsilon}$$

$$\frac{1}{\rho}\frac{dE_z}{d\phi} = H_\rho\left(\frac{\omega^2\mu\varepsilon+\gamma^2}{j\omega\varepsilon}\right)$$

$$H_\rho = \frac{j\omega\varepsilon}{h^2}\frac{1}{\rho}\frac{dE_z}{d\phi} \rightarrow (11)$$

The boundary condition requires that the tangential component of electric field E_z at $\rho = a$ Vanishes

$\therefore$ from eq (1)

$$E_z = C_0 J_n(\rho h)\cos n\phi\, e^{-\gamma z} \rightarrow (1)$$

$$\therefore\ 0 = C_0 J_n(ah)\cos n\phi\, e^{-\gamma z}$$

$$\therefore\ J_n(ah) = 0$$

Since $J_n(ah)$ are oscillator functions, there are infinite number of roots of $J_n(ah)$. The values of these roots for which $J_n(ah)=0$ are called eigen values and are denoted by P_{nm}, were.

then $ah = P_{nm}$

$$h = \frac{P_{nm}}{a} \rightarrow (12)$$

P_{nm} values for TM_{nm} mode in circular waveguide

n \ m	1	2	3
0	2.405	5.52	8.645
1	3.832	7.106	10.173
2	5.135	8.417	11.620
3	6.380	9.761	13.05

∴ Therefore from eq ①

$$E_z = C_0 \, J_n(\rho h) \cos n\phi \, e^{-\gamma z}$$

$$E_z = C_0 \, J_n\left(\frac{P_{nm}}{a}\rho\right) \cos n\phi \, e^{-\gamma z} \rightarrow (13)$$

From eq ⑧ $E_\rho = \frac{-\gamma}{h^2} \frac{dE_z}{d\rho}$

$$E_\rho = \frac{-\gamma}{h^2} C_0 \, J_n'\left(\frac{P_{nm}}{a}\rho\right) \cos n\phi \, e^{-\gamma z} \rightarrow (14)$$

From eq ⑨ $H_\phi = \frac{-j\omega\varepsilon}{h^2} \frac{dE_z}{d\rho}$

$$H_\phi = \frac{-j\omega\varepsilon}{h^2} C_0 \cdot J_n'\left(\frac{P_{nm}}{a}\rho\right) \cos n\phi \, e^{-\gamma z} \rightarrow (15).$$

from eq ⑩ $E_\phi = \frac{-\gamma}{h^2} \cdot \frac{1}{\rho} \frac{dE_z}{d\phi}$

$$E_\phi = \frac{\gamma}{h^2} \frac{1}{\rho} C_0 \, J_n\left(\frac{P_{nm}}{a}\rho\right) n \sin n\phi \, e^{-\gamma z} \rightarrow (16)$$

from eq ⑪

$$H_\rho = \frac{-j\omega\varepsilon}{h^2} \frac{1}{\rho} C_0 \, J_n\left(\frac{P_{nm}}{a}\rho\right) n \sin n\phi \, e^{-\gamma z} \rightarrow (17).$$

cutoff wave length (λ_c):

cut off wave length is that the wavelength at which the propagation constant γ vanishes.

$$h^2 = \gamma^2 + \omega^2\mu\varepsilon$$

at $\omega = \omega_c$, $\gamma = 0$

$$h^2 = \omega_c^2\mu\varepsilon \rightarrow ①$$

$$h = \omega_c\sqrt{\mu\varepsilon}$$

$$\omega_c = \frac{h}{\sqrt{\mu\varepsilon}} = hc$$

$$2\pi f_c = hc \qquad \left[c = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \right]$$

$$f_c = \frac{hc}{2\pi}$$

$$\lambda_c = \frac{c}{f_c} = \frac{c}{(hc/2\pi)} = \frac{2\pi}{h}$$

$$\boxed{\therefore \lambda_c = \frac{2\pi}{h}}$$

for TE_{nm} wave

$$h = \frac{P'_{nm}}{a}$$

$$\lambda_{c_{TE}} = \frac{2\pi}{h} = \frac{2\pi}{P'_{nm}/a} = \frac{2\pi a}{P'_{nm}} \rightarrow (2)$$

for TM_{nm} wave

$$\therefore h = \frac{P_{nm}}{a}$$

$$\lambda_{c_{TM}} = \frac{2\pi a}{P_{nm}} \longrightarrow (3)$$

<u>phase constant</u>:

$[h^2 = \gamma^2 + \omega^2 \mu \varepsilon]$

$$\gamma = \alpha + j\beta$$

$$= \sqrt{-\omega^2 \mu\varepsilon + h^2}$$

when wave is propagating $\gamma = j\beta$

$$j\beta = \sqrt{h^2 - \omega^2 \mu\varepsilon}$$

$$-\beta^2 = h^2 - \omega^2 \mu\varepsilon$$

$$\beta = \sqrt{\omega^2 \mu\varepsilon - h^2}$$

from eq ①

$$\beta = \sqrt{\omega^2 \mu \varepsilon - \omega_c^2 \mu \varepsilon}$$

$$\beta = \sqrt{\mu \varepsilon (\omega^2 - \omega_c^2)} \rightarrow ④$$

phase velocity: (v_p) $v_p = \frac{\omega}{\beta}$

$$v_p = \frac{\omega}{\sqrt{\mu\varepsilon(\omega^2-\omega_c^2)}} = \frac{c}{\sqrt{\left(1 - \frac{\omega_c^2}{\omega^2}\right)}}$$

$$v_p = \frac{c}{\sqrt{\left(1 - f_c^2/f^2\right)}}$$

Group velocity (v_g):

$$v_g = \frac{d\omega}{d\beta} = c\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

we know $\beta = \sqrt{\mu\varepsilon(\omega^2 - \omega_c^2)}$

$$\frac{d\beta}{d\omega} = \frac{1}{2\sqrt{\mu\varepsilon(\omega^2-\omega_c^2)}} \times 2\omega\mu\varepsilon = \frac{\omega\varepsilon\mu}{\sqrt{\mu\varepsilon(\omega^2-\omega_c^2)}}$$

$$= \frac{\omega\sqrt{\mu\varepsilon}}{\sqrt{\omega^2 - \omega_c^2}}$$

$$\frac{d\beta}{d\omega} = \frac{1}{c\sqrt{1 - \frac{\omega_c^2}{\omega^2}}}$$

but $v_g = \frac{d\omega}{d\beta} = c\sqrt{1 - \frac{\omega_c^2}{\omega^2}}$

$$= c\sqrt{1 - \frac{f_c^2}{f^2}}$$

Guided wavelength: $\lambda_g = \dfrac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}} = \dfrac{\lambda_0}{\sqrt{1-\left(f_c/f\right)^2}}$

Dominant mode:

The mode which is having lowest cutoff frequency is called dominant mode

$\lambda_c = \dfrac{2\pi a}{P'_{nm}}$

for TM wave $\lambda_c = \dfrac{2\pi a}{P_{nm}}$

$P_{nm} \rightarrow$ $h_{ny} = 2.405$ i.e TM_{01} mode by long Hand values. TM_{01} is dominant mode

from the P'_{nm} table, for TE_{11} mode.

$P'_{nm} = 1.841$ which is the lowest value

$\therefore$ TE_{11} mode is dominant mode circular waveguides.

Degenerate mode: The modes which are having same cut off frequency are called degenerate modes.

from the P'_{nm} and P_{nm} table.

$P'_{nm} = P_{nm}$.

Hence all the TE_{0m} and TM_{1m} modes are degenerate.

Wave impedance (Z_z):

The wave impedance is defined as the ratio of strength of electric field in one transverse direction to the strength of magnetic field along other transverse direction.

$$Z_z = \frac{E_\rho}{H_\phi}$$

for TE wave

$$Z_{zTE} = \frac{E_\rho}{H_\phi}$$

$$= \frac{\frac{-j\omega\mu}{h^2}\cdot\frac{1}{\rho}\cdot\frac{dH_z}{d\phi}}{\frac{-\gamma}{h^2}\cdot\frac{1}{\rho}\cdot\frac{dH_z}{d\phi}} = \frac{j\omega\mu}{\gamma}$$

when the wave is propagating $\gamma = j\beta$.

$$Z_{zTE} = \frac{j\omega\mu}{j\beta} = \frac{\omega\mu}{\beta}$$

$$= \frac{\omega\mu}{\sqrt{\mu\varepsilon(\omega^2-\omega_c^2)}} = \frac{\eta}{\sqrt{1-(f_c/f)^2}}$$

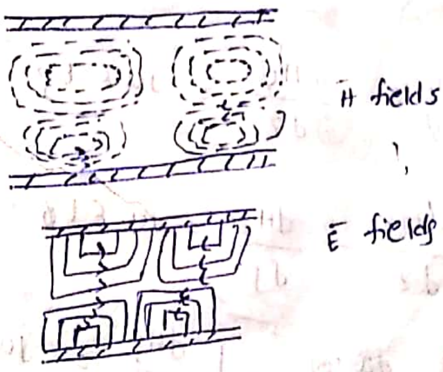
for TM wave

$$Z_{zTM} = \frac{E_\rho}{H_\phi} = \frac{-\frac{\gamma}{h^2}\frac{dE_z}{d\rho}}{\frac{-j\omega\varepsilon}{h^2}\frac{dE_z}{d\rho}} = \frac{\gamma}{j\omega\varepsilon} = \frac{j\beta}{j\omega\varepsilon}$$

$$Z_{TM} = \frac{\beta}{\omega\varepsilon} = \eta\sqrt{1-(f_c/f)^2}$$

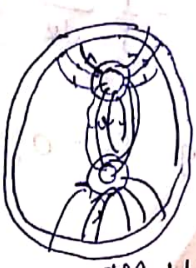
The mode field configuration of a circular waveguide are shown in figure below.

(a) $TE_{0,1}$ mode



(b). $TM_{0,1}$ mode

(C) $TE_{1,1}$ mode



$TM_{1,1}$ mod.

(C) $TE_{2,1}$ mode

TEM modes in circular Waveguides:

TEM mode is characterised by $E_z = H_z = 0$

The maxwell's equations are

$$\nabla \times E = -j\omega\mu \bar{H}$$

$$\nabla \times E = \frac{1}{\rho}\begin{vmatrix} \bar{i} & \rho\bar{j} & \bar{k} \\ \frac{d}{d\rho} & \frac{d}{d\phi} & \frac{d}{dz} \\ E_\rho & \rho E_\phi & E_z \end{vmatrix} = -j\omega\mu\left(H_\rho\bar{i} + H_\phi\bar{j} + H_z\bar{k}\right)$$

$$\frac{1}{\rho}\frac{dE_z}{d\phi} - \frac{dE_\phi}{dz} = -j\omega\mu H_\rho$$

$$\frac{dE_z}{d\rho} - \frac{dE_\rho}{dz} = j\omega\mu H_\phi$$

$$\frac{1}{\rho}\frac{d}{d\rho}(\rho E_\phi) - \frac{1}{\rho}\frac{dE_\rho}{d\phi} = -j\omega\mu H_z$$

$$\nabla \times H = j\omega\epsilon E$$

$$\nabla \times H = \frac{1}{\rho}\begin{vmatrix} \bar{i} & \rho\bar{j} & \bar{k} \\ \frac{d}{d\rho} & \frac{d}{d\phi} & \frac{d}{dz} \\ H_\rho & \rho H_\phi & H_z \end{vmatrix} = j\omega\epsilon\left[E_\rho\bar{i} + E_\phi\bar{j} + E_z\bar{k}\right]$$

$$\frac{1}{\rho}\frac{dH_z}{d\phi} - \frac{dH_\phi}{dz} = j\omega\epsilon E_\rho$$

$$\frac{dH_\rho}{dz} - \frac{dH_z}{d\rho} = j\omega\epsilon E_\phi$$

$$\frac{1}{\rho}\frac{d(\rho H_\phi)}{d\rho} - \frac{1}{\rho}\frac{dH_\rho}{d\phi} = j\omega\epsilon E_z$$

Substituting $E_z = 0$, $H_z = 0$

$$\gamma E_\phi = -j\omega\mu H_\rho \rightarrow (a)$$

$$\gamma E_\rho = j\omega\mu H_\phi \rightarrow (b)$$

$$\frac{d}{d\rho}(\rho E_\phi) = \frac{dE_\rho}{d\phi} \rightarrow (c)$$

$$\gamma H_\phi = j\omega\varepsilon E_\rho \rightarrow (d)$$

$$-\gamma H_\rho = j\omega\varepsilon E_\phi \rightarrow (e)$$

$$\frac{d(\rho H_\phi)}{d\rho} = \frac{dH_\rho}{d\phi} \rightarrow (f)$$

fom (a) $\frac{H_\rho}{E_\phi} = \frac{-\gamma}{j\omega\mu}$

from (e) $\frac{H_\rho}{E_\phi} = \frac{-j\omega\varepsilon}{\gamma}$

(a) = (e)

$$\frac{-\gamma}{j\omega\mu} = \frac{-j\omega\varepsilon}{\gamma}$$

$$\gamma^2 = -\omega^2\mu\varepsilon$$

wave propagation $\gamma = j\beta$

$$-\beta^2 = -\omega^2\mu\varepsilon$$

$$\beta = \omega\sqrt{\mu\varepsilon} \rightarrow (1)$$

for a circular wave guide the phase constant is

$$\beta = \sqrt{\omega^2\mu\varepsilon - h^2} \rightarrow (2)$$

from (1) & (2)

$$\sqrt{\omega^2\mu\varepsilon - h^2} = \omega\sqrt{\mu\varepsilon}$$

$$\omega^2\mu\varepsilon - h^2 = \omega^2\mu\varepsilon$$

$$h = 0$$

∴ therefore the cut off frequency, $\lambda_c = \frac{2\pi}{h} = \infty$

$$f_c = \frac{c}{\lambda_c} = 0$$

This implies that waveguide passes all the frequency.

phase velocity $= \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}}$

which is the velocity of light in unbounded dielectric.

wave impedance $\eta = \frac{E_r}{H_\phi} = \frac{\gamma}{j\omega\varepsilon} = \frac{j\beta}{j\omega\varepsilon}$

$$\eta = \frac{\beta}{\omega\varepsilon} = \frac{\omega\sqrt{\mu\varepsilon}}{\omega\varepsilon} = \sqrt{\frac{\mu}{\varepsilon}}$$

which is the impedance of a loss less transmission line in a dielectric.

From the above properties, it is clearly shows that all the parameters are equal to the parameters of two conductor lossless transmission line. This implies that TEM mode can exist only in two conductor system. But the wave guide is a single conductor system. therefore TEM mode cannot exist in circular waveguide.

UNIT-VI

CIRCULAR WAVE GUIDES

Assignment-Cum-Tutorial Questions

SECTION-A

1. Dominant mode in circular wave guide is given by [ ]
   a) TE_{10} b) TE_{11} c) TE_{01} d) TE_{12}
2. In cylindrical waveguide, the impedance, Z_{TE} is given by [ ]
   a) $\beta/\omega\mu$ b) $\omega\mu/\beta$ c) $\omega\beta/\mu$ d) $\omega\mu\beta$
3. Possible number of modes that can exist in cylindrical waveguides is [ ]
   a) Zero b) One c) 2 d) Infinite
4. Nonexistent modes in circular wave guides are [ ]
   a) TE_{10} b) TE_{00} c) both d) None of these
5. The cut off wavelength of the guided wave is [ ]
   (a) $f_c = \frac{2a}{m}$ (b) $f_c = \frac{m}{2a\sqrt{\mu\varepsilon}}$ (c) $f_c = \frac{a}{2m\sqrt{\mu\varepsilon}}$ (d) None
6. The Dominant mode in the circular waveguide is [ ]
   A) TM_{11} B) TE_{11} C) TM_{10} D) TE_{10}
7. The waveguide is a _____________ filter. [ ]
   A) Low pass B) High pass C) Band pass D) Band stop
8. Degenerative modes are the modes having [ ]
   A) Equal cut-off frequencies B) Equal tangential powers
   C) Equal wave impedances D) Equal phase velocities
9. Theoretically, number of modes that can exists in cylindrical waveguides is [ ]
   A) zero B) one C) two D) infinite
10. The cut-off frequency in circular waveguides for *TM* mode is ---------
11. The cut-off frequency in circular waveguides for *TE* mode is ----------
12. The cut-off wavelength in circular waveguides for *TM* mode is ---------
13. The cut-off wavelength in circular waveguides for *TE* mode is ---------
14. The phase velocity in waveguides is --------- whereas the group velocity is -------
15. In waveguides, the wave impedance is ------ for *TE* mode and it is ------ for *TM* mode.
16. Reflective attenuation comes into being when wave frequency is----- than cut-off frequency.
17. Dissipative attenuation comes into being when wave frequency is----- than cut-off frequency.
18. The dominant mode in circular waveguides is ________
19. The expression for phase velocity in circular waveguides is ________
20. The expression for guide wavelength of circular waveguide is________
21. The expression for wave impedance in TE mode of circular waveguide is_________
22. The cutoff frequency in circular waveguide for TM mode is ________
23. The cutoff wavelength in circular waveguide for TE mode is _______
24. How TE and TM modes are defined in circular waveguides?

25. Explain the filter characteristics of circular waveguide.
26. Dominant mode in rectangular guides is—— whereas in circular waveguides it is ——
27. Degenerate modes of waveguides are—— modes having same cut-off frequency.
28. The order of mode subscripts *m* and *n* in rectangular guides is—— whereas in circular guides it is——————.

SECTION-B

Descriptive questions

1. Describe the modal propagation characteristics in the circular waveguides. How the mode description of CWGs is different from that of RWGs. **[C04]**
2. Define different types of impedances of the waveguides and write down expressions for the impedances in case of CWGs. **[C04]**
3. What are two types of attenuations that exist in CWGs? Write down general expressions for both the types of attenuation. **[C04]**
4. Derive the field components of TM waves in circular waveguides. **[C05]**
5. Derive the field components of TE waves in circular waveguides. **[C05]**
6. TEM mode is not possible in circular waveguide. Justify the statement. **[C05]**
7. Illustrate the field patterns of TM_{01} and TE_{01} modes in circular waveguides with neat sketches. **[C05]**
8. State the formulas for the following parameters related to air filled circular waveguide operated in TE_{mn} mode: **[C05]**
   i. Cutoff frequency
   ii. Propagation constant
   iii. Wavelength in the waveguide
   iv. Phase velocity
9. Define dominant and degenerative modes. Write examples for few degenerative modes in circular waveguides. **[C04]**
10. In circular waveguides, TE_{10} mode is not possible. Justify this? **[C05]**
11. Which is the dominant mode in circular waveguides? Why TE_{10} mode is not possible in circular waveguides? **[C05]**
12. What is cut off frequency? Derive relation connecting cut off frequency with dimensions of guide. **[C04]**

Problems

1. An air-filled circular waveguide has radius of 5cm and act as being operated at 3GHz. Find its cut-off frequency, cut-off wavelength, guide wavelength and phase velocity for (a) TE_{11} and (b)TM_{01}. **[C06]**
2. An air-filled circular waveguide with an inner radius of 1.2 cm is operating in TM_{01}mode. Determine its cutoff frequency. If it is operating at a frequency of 10GHz, then find wavelength in the waveguide. **[C06]**
3. TE_{11} wave is propagating through an air filled circular waveguide of diameter 8 cm. the first order Bessel root value for this mode Xnp = ha = 1.841. Then find:
   i) Cutoff frequency ii) Wave impedance. **[C06]**
4. A TE_{11} wave is propagating through a circular wave guide has a diameter. The diameter of the guide is 10 cm, and the guide is air filled. Compute the following:

i. Cutoff frequency
ii. Wavelength in the waveguide
iii. Phase constant
iv. Wave impedance. **[C06]**

5. An air filled circular waveguide is to have dimensions such that fc = 0.6f for TE_{11} mode and is to be operated at 4 GHz. **[C06]**
   1. Determine i) diameter of the waveguide ii) guide wavelength
6. Find i) cutoff wavelength, ii) cutoff frequency, iii) wavelength in the guide for the dominant mode of operation in an air filled circular waveguide of inner diameter 6 cm. **[C06]**
7. A TE_{11} wave is propagating through a circular waveguide. If the guide is air filled and the diameter of the guide is 8 cm. Find i) cutoff frequency, ii) guide wavelength for frequency of 4 GHz, and iii) the wave impedance. **[C06]**
8. An air filled circular waveguide having radius of 5 cm is being operated at 3 GHz. Find its i) cut-off frequency, ii) cut-off wavelength and iii) phase velocity for TE_{11} mode. **[C06]**
9. An air filled circular waveguide having radius of 1.2 cm is being in TM_{01} mode. Determine its i) cut-off frequency, ii) guide wavelength and iii) phase velocity at 10 GHz. **[C06]**
10. An air filled circular waveguide having radius of 8 cm is being operated at 4 GHz. Find its i) cut-off frequency, ii) cut-off wavelength and iii) phase velocity for TM_{01} mode. **[C06]**

SECTION-C

1. Degenerative modes in circular waveguides are [ ]

   A) TE_{01} and TM_{11} B) TM_{01} and TE_{11}
   C) TE_{22} and TM_{22} D) TE_{10} and TE_{01}

2. The guide wavelength $\bar{\lambda}$ is given by [ ]

   (a) $\bar{\lambda} = 2\pi / \bar{\beta}$ (b) $\bar{\lambda} = 2\pi \Big/ \sqrt{\omega^2 \mu\varepsilon - (m\pi/a)^2}$
   (c) $\bar{\lambda} = 2\pi \Big/ \omega\sqrt{\mu\varepsilon}\sqrt{1+(\lambda/\lambda_c)^2}$ (d) All

3. The phase shift constant $\bar{\beta}$ of the guided wave is [ ]

   (a) $\bar{\beta} = \sqrt{\omega^2 \mu\varepsilon - (m\pi/a)^2}$ (b) $\bar{\beta} = \omega\sqrt{\mu\varepsilon}\sqrt{1+(f_c/f)^2}$
   (c) $\bar{\beta} = \omega\sqrt{\mu\varepsilon}\sqrt{1+(\lambda/\lambda_c)^2}$ (d) All.

4. The cut off frequency of the guided wave is [ ]

   (a) $f_c = \frac{2a}{m}$ (b) $f_c = \frac{m}{2a\sqrt{\mu\varepsilon}}$ (c) $f_c = \frac{a}{2m\sqrt{\mu\varepsilon}}$ (d) None